Math 2100 – Homework 4

Fall 2022

due Wednesday, **November 2**, at the start of class

Sections 2.3, 2.4, 2.5, some 3.1

This homework assignment was written in LaTeX. You can find the source code on the course website.

Instructions: This assignment is due at the *beginning* of class. Please write the questions in the correct order. Explain all reasoning.

1. Use induction to prove that for all integers $n \geq 0$, the quantity $2^{2n+1} + 5^{2n+1}$ is divisible by 7.
2. Prove that $1^3 + 2^3 + \cdots + n^3 = (1 + 2 + \cdots + n)^2$.
3. Prove that there exists a positive integer n such that $\frac{1}{n \ln(n)} < 0.0001$.
4. Prove that any real number r that makes the equation $r - \dfrac{1}{r} = 5$ true must be irrational.
5. Prove that at a completely full Milwaukee Bucks game at the Fiserv Forum, there *must* be at least two people that have both the same birthday *and* the same first initial. (Note: you will have to look up the capacity of the arena!)
6. Prove that if $a + b + c \geq 35$, then either $a \geq 10$, $b \geq 12$, or $c \geq 13$.
7. Use the pigeonhole principle to prove that given any five integers, there will be two that have a sum or difference divisible by 7.
8. Prove that if any five points other than $(0, 0)$ are placed on the coordinate plane, then there are two points, call them A and B, such that the angle formed by the rays from $(0, 0)$ to A and from $(0, 0)$ to B is acute.
9. ~~Write each of the following sets in set-builder notation.~~
   (a) ~~The set S of integers that are multiples of 3 and a prefect square.~~
   (b) ~~The set T of positive integers that are bigger than 10 and whose ones digit is a 5.~~
   (c) ~~The set R of real numbers whose square is a rational number.~~
10. ~~List five elements in each of the following sets, unless there are fewer than 5 elements in the set (in which case, justify how you know you've listed all of the elements).~~
    (a) ~~$A = \{x \in \mathbb{R} : x^2 \in \mathbb{N}\}$~~
    (b) ~~$B = \{S \subseteq \{1, 2, 3, 4\} : \text{the sum of the elements of } S \text{ is even}\}$~~
    (c) ~~$C = \{q \in \mathbb{N} : q = 2k \text{ for some } k \in \mathbb{N} \text{ and } q = 2\ell + 1 \text{ for some } \ell \in \mathbb{N}\}$~~