MATH 2100 – HOMEWORK 3

Fall 2022

due Wednesday, **October 19**, at the start of class

Sections 2.1, 2.2, 2.3, 2.4

This homework assignment was written in LaTeX. You can find the source code on the course website.

Instructions: This assignment is due at the *beginning* of class. Please write the questions in the correct order. Explain all reasoning.

1. Decide if the following statement is true or false. If it's true, prove it. If it's false, provide a counterexample.

   If x, y, and z are integers and if x divides y and x divides z, then x^2 divides yz.

2. Decide if the following statement is true or false. If it's true, prove it. If it's false, provide a counterexample.

   If x, y, and z are integers and if x divides z and y divides z, then xy divides z.

3. Decide if the following statement is true or false. If it's true, prove it. If it's false, provide a counterexample.

   If n is a positive even integer, then $3^n + 1$ is divisible by 5.

4. Decide if the following statement is true or false. If it's true, prove it. If it's false, provide a counterexample.

   If n is a positive even integer, then $n^3 + 2n$ is divisible by 4.

5. Decide if the following statement is true or false. If it's true, prove it. If it's false, provide a counterexample.

   If m is a positive odd integer, then $m^2 - 1$ is divisible by 8.

6. Prove that if 3 divides $4^{n-1} - 1$ then 3 divides $4^n - 1$.

7. Prove that no perfect square can have the form $3n + 2$ for an integer n.

8. Prove that for all positive integers n,

$$\sum_{k=0}^{n} (k \cdot k!) = (n+1)! - 1.$$

9. Prove that for all positive integers $n \geq 2$, the number $2^{3n} - 1$ is not prime. (Hint: do some experimenting to figure out a more specific fact about how the numbers $2^{3n} - 1$ factor, and then use induction to prove that fact.)

10. Prove that for all positive integers $n \geq 4$,

$$n! > 2^n.$$