MATH 2100 – EXAM 2

Friday, October 28

Name: ____

Instructions: Please write your work neatly and clearly. **You must explain all reasoning. It is not sufficient to just write the correct answer.** You have 50 minutes to complete this exam. You may not use calculators, notes, or any other external resources.

<u>Scores</u>

1	
2	
3	
4	
5	
6	

The Marquette University honor code obliges students:

- To fully observe the rules governing exams and assignments regarding resource material, electronic aids, copying, collaborating with others, or engaging in any other behavior that subverts the purpose of the exam or assignment and the directions of the instructor.
- To turn in work done specifically for the paper or assignment, and not to borrow work either from other students, or from assignments for other courses.
- To complete individual assignments individually, and neither to accept nor give unauthorized help.
- To report any observed breaches of this honor code and academic honesty.

If you understand and agree to abide by this honor code, sign here:

____

1. Suppose that every college basketball team plays 32 games in a season. (In reality, it's usually fewer.) Assuming that no games end in a tie, prove that there are at least three of the 68 teams in the March Madness tournament that have the exact same win-loss record.

2. Prove that for all positive integers n,

$$2 \cdot \left(\sum_{i=1}^{n} 3^{i-1} \right) = 3^n - 1.$$

3. Determine if the following statement is true. If it is, prove it. If not, give a counterexample.

If a divides $x \cdot y$, then a divides x or a divides y.

4. Use induction to prove that for any positive integer n, the number $8^n - 3^n$ is a multiple of 5.

5. Determine if the following statement is true. If it is, prove it. If not, give a counterexample.

> Prove that if W is an integer that is not divisible by 5, then W^2 is either one more than a multiple of five or is four more than a multiple of five.

6. Use a proof by contradiction to show that there is no largest perfect square.