MATH 2100 – EXAM 1

Friday, September 30

Name: _____

Instructions: Please write your work neatly and clearly. **You must explain all reasoning. It is not sufficient to just write the correct answer.** You have 50 minutes to complete this exam. You may not use calculators, notes, or any other external resources.

Scores

1	
2	
3	
4	
5	
6	

The Marquette University honor code obliges students:

- To fully observe the rules governing exams and assignments regarding resource material, electronic aids, copying, collaborating with others, or engaging in any other behavior that subverts the purpose of the exam or assignment and the directions of the instructor.

- To turn in work done specifically for the paper or assignment, and not to borrow work either from other students, or from assignments for other courses.

- To complete individual assignments individually, and neither to accept nor give unauthorized help.

- To report any observed breaches of this honor code and academic honesty.

If you understand and agree to abide by this honor code, sign here:

_____

1. Use a truth table to determine whether the following logical equivalence is true:

$$(r \to p) \land (\neg q \to r) \equiv p \lor \neg r$$

2. Consider the implication

$$C = \text{"If it is raining, then your car's headlights are on and your windows are closed."}$$

(a) What is the converse of C?

(b) What is the contrapositive of C?

(c) What is the inverse of C?

(d) Which of the above 3 statements is/are logically equivalent to C?

3. Consider the statement

Every city has a resident who loves the weather every day.

(a) Convert the statement from English to math using sets, quantifiers, and predicates. Your predicate should have *no quantifiers in it*.

(b) Negate the statement mathematically.

(c) Convert the negation back to English.

4. (a) Convert the statement below to English. Then decide whether it's true or false, with full explanation of your justification.

$$\forall x \in \mathbb{R}, (x > 0 \rightarrow (\exists y \in \mathbb{R}, 0 < y < x)).$$

(b) Convert the statement below to mathematical symbols. Then decide whether it's true or false, with full explanation of your justification.

For every natural number x there is a natural number y for which xy = 1.

5. The map below shows a region consisting of six countries.
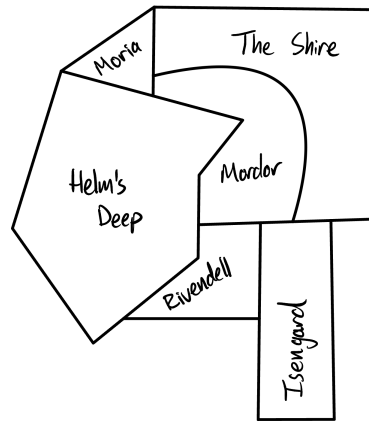
(a) Draw a graph whose vertices represent the countries with an edge between two counties if they share a border.

(b) Show that it is possible to use four colors to color the vertices of this graph such that no two vertices that are connected by an edge have the same color.

(c) Describe why it NOT is possible to use three colors to color the vertices of this graph such that no two vertices that are connected by an edge have the same color. (Don't just show that one attempt at using three colors fails, justify that every attempt must fail.)

(d) What do the parts above tell you about coloring the original map?

6. Give a full formal proof by contrapositive of the statement:

If $m + n$ is odd, then m is odd or n is odd.