Friday, Dec. 9 – Fall '22
Lecture #41

Announcements / Reminders

* Practice Wiley Plus (5.2, 5.3) until Sunday
* Course Evaluations are open. If 90% of the class does them, I will give everyone two bonus points.

* Final Exam, 1pm–3pm on Monday, in this room
* ODS proctoring – check email for room #

Section 5.3 – The Fundamental Theorem of Calculus and Interpretations

Last time:

" $\displaystyle\int_a^b f(t)\,\underbrace{dt}$ " means " the (signed) area between the x-axis and $f(t)$ between $t=a$ and $t=b$ "

just a bit of notation

"dt" kind of stands for
"a small change in t"
like Δt.

an infinitely small quantity

$\frac{dy}{dx}$

In terms of units, $f(t)$ might be
something like "feet/sec"

$\boxed{\int f(t)\,dt}$ = ← has a unit of feet

$\frac{\text{feet}}{\cancel{\text{sec}}} \cdot \cancel{\text{sec}} = \text{feet}$

Summary:

$\int_a^b v(t)\,dt$ = area under the curve $v(t)$
from $t=a$ to $t=b$
= change in position from $t=a$
to $t=b$

If $s(t)$ is the position function
$= s(b)-s(a)$

We know that $v(t) = s'(t)$.

This gives us a hint about how to calculate integrals. To calculate $\int_a^b f(t)\,dt$ find a different function $F(t)$, whose derivative is $f(t)$, and do $F(b) - F(a)$.

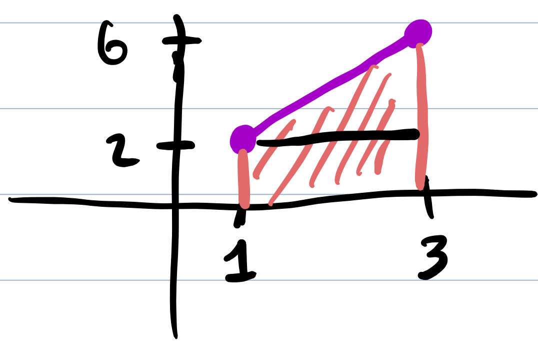
$$F'(t) = f(t)$$

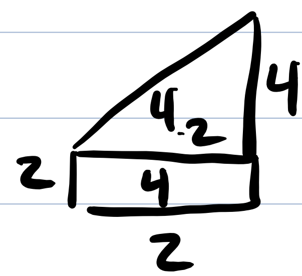
Fundamental Theorem of Calculus

If f is continuous on the interval $[a,b]$, and if $f(t) = F'(t)$, then

$$\int_a^b f(t)\,dt = F(b) - F(a)$$

<u>Ex:</u> $\int_1^3 2x\,dx$ without FTC





$= 4+4$
$= 8$

Ex: $\int_1^3 2x\,dx$ with FTC

We want to find $F(x)$ whose derivative is $2x$.

$F(x) = x^2$ works

$$\int_1^3 2x\,dx = F(3) - F(1) = 3^2 - 1^2 = 9 - 1 = 8 \checkmark$$

Couldn't we have used $F(x) = x^2 + 100$?

$$\int_1^3 2x\,dx = F(3) - F(1) = (3^2 + \cancel{100}) - (1^2 + \cancel{100})$$
$$= 9 - 1 = 8 \checkmark$$

"$+C$"

Ex: Let $F(t)$ represent a bacterial population which is 5 million at $t = 0$. After t hours, the population is growing at an instantaneous rate of 2^t million bacteria per hour. What is the population at $t = 1$?

The underlined part is telling us that $F'(t) = 2^t$.

~~($F'(1) = 2$ $F'(2) = 4$~~

Know $F(0) = 5$

$F'(t) = 2^t$

Want $F(1)$.

We want "change in population from $t=0$ to $t=1$."

FTC: $F(1) - F(0) = \int_0^1 2^t \, dt$

with a calculator: ≈ 1.44

$F(1) - 5 \approx 1.44$

$F(1) \approx 6.44$ million

without a calculator:

$$\int_0^1 2^t \, dt = \frac{2}{\ln(2)} - \frac{1}{\ln(2)} \approx 1.44\ldots\ldots$$

we want a function $F(t)$ whose derivative is 2^t.

$$(2^t)' = \ln(2)\cdot 2^t$$

$$\left(\frac{2^t}{\ln(2)}\right)' = \frac{1}{\ln(2)}\cdot(2^t)' = \frac{1}{\cancel{\ln(2)}}\cdot\cancel{\ln(2)}\cdot 2^t$$

$$= 2^t$$

Ex:

$$\int_0^{2\pi} \cos(\theta)\,d\theta = \sin(2\pi) - \sin(0) = 0-0$$
$$= 0$$

Need a function whose derivative is $\cos(\theta)$.

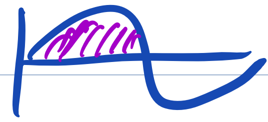
$\sin(\theta)$

$\cos(\theta)$

positive and negative area cancels out to give 0.

Group Work:

$-\cos(\theta)$



1) $$\int_0^{\pi} \sin(\theta)\, d\theta = -\cos(\pi) - (-\cos(0))$$
$$-(-1) + 1 = 2 \checkmark$$

2) $$\int_{-3}^{-2} e^t\, dt = e^{-2} - e^{-3}$$

e^t

3) $$\int_{-1}^{2} t^2\, dt = \frac{1}{3}(2)^3 - \frac{1}{3}(-1)^3$$

$\frac{1}{3}t^3$

$$= \frac{8}{3} + \frac{1}{3} = 3$$