Monday, Dec 2 - Fall '22
Lecture #38

5.3

Announcements / Reminders

* Wiley Plus #14 due Wed. night
* Quiz 12 in discussion on Thursday

(4.7, 5.1)

* ODS - Final exam scheduling deadline

* Course Evaluations are open

Today: Finish 5.1, go back and do 4.7.

* Final Exam is the Monday of Exam Week, 1pm-3pm

Suppose you're driving a car and as you're speeding up, you look down at the speedometer every 2 seconds and write down your speed.

time (sec)	0	2	4	6	8	10
speed (ft/sec)	20	30	38	44	48	50

Can you tell how far you traveled?

* Between $t=0$ and $t=2$, you traveled <u>at least</u> $2 \cdot 20 = 40$ feet
* Between $t=2$ and $t=4$, you traveled at <u>least</u> $2 \cdot 30 = 60$ ft.

Overall: $\underbrace{2\cdot 20}_{0\to 2} + \underbrace{2\cdot 30}_{2\to 4} + \underbrace{2\cdot 38}_{4\to 6} + \underbrace{2\cdot 44}_{6\to 8} + \underbrace{2\cdot 48}_{8\to 10}$

$= 360$ feet

* must be an <u>underestimate</u> because you're always speeding up *

Overestimate: Use the right value of each 2 second window 3

$0 \to 2$ $2 \to 4$ $4 \to 6$ $6 \to 8$ $8 \to 10$

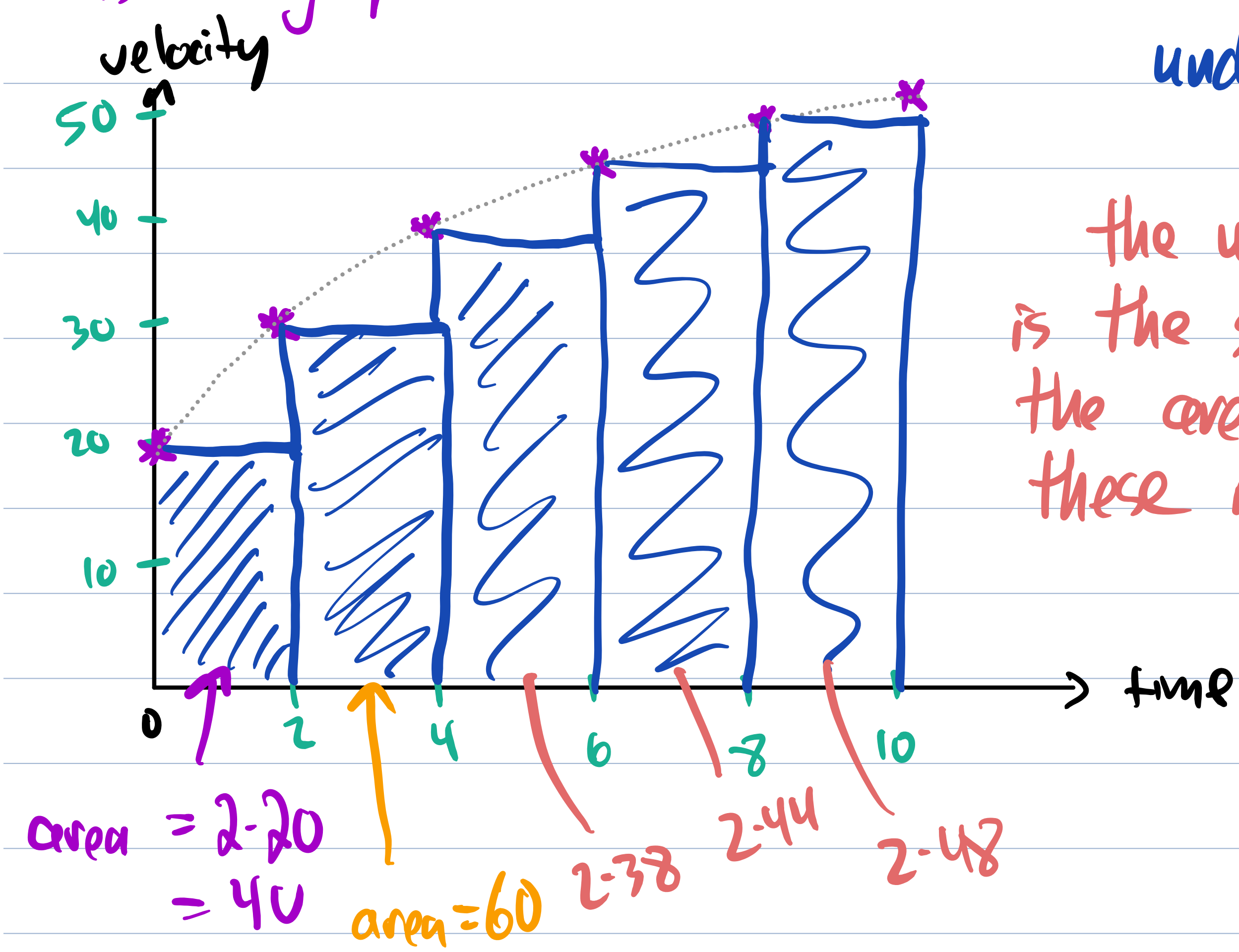
$2 \cdot 30 + 2 \cdot 38 + 2 \cdot 44 + 2 \cdot 48 + 2 \cdot 50$

$= 420$ feet

More accurate data (example: every 1 second) ⇝ better estimates

time (sec)	0	2	4	6	8	10
speed (ft/sec)	20	30	38	44	48	50

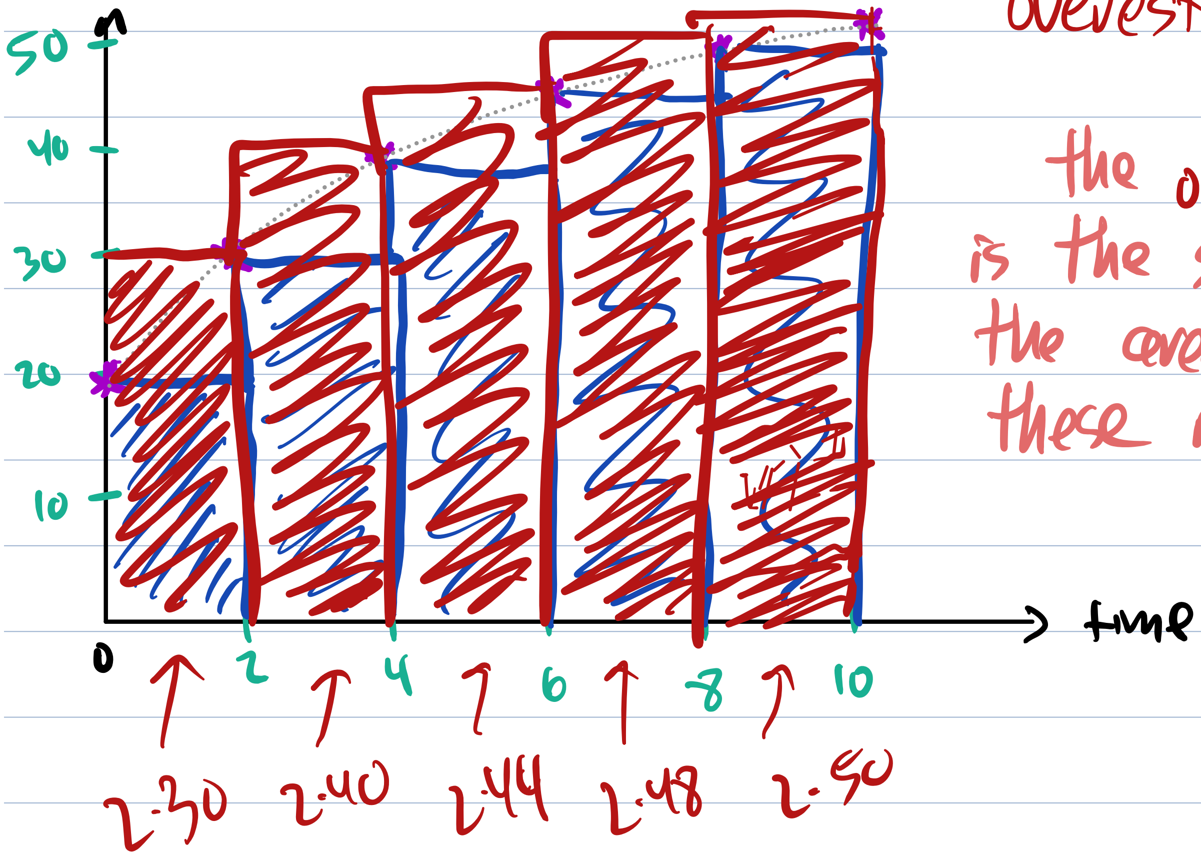
As a graph:



underestimate

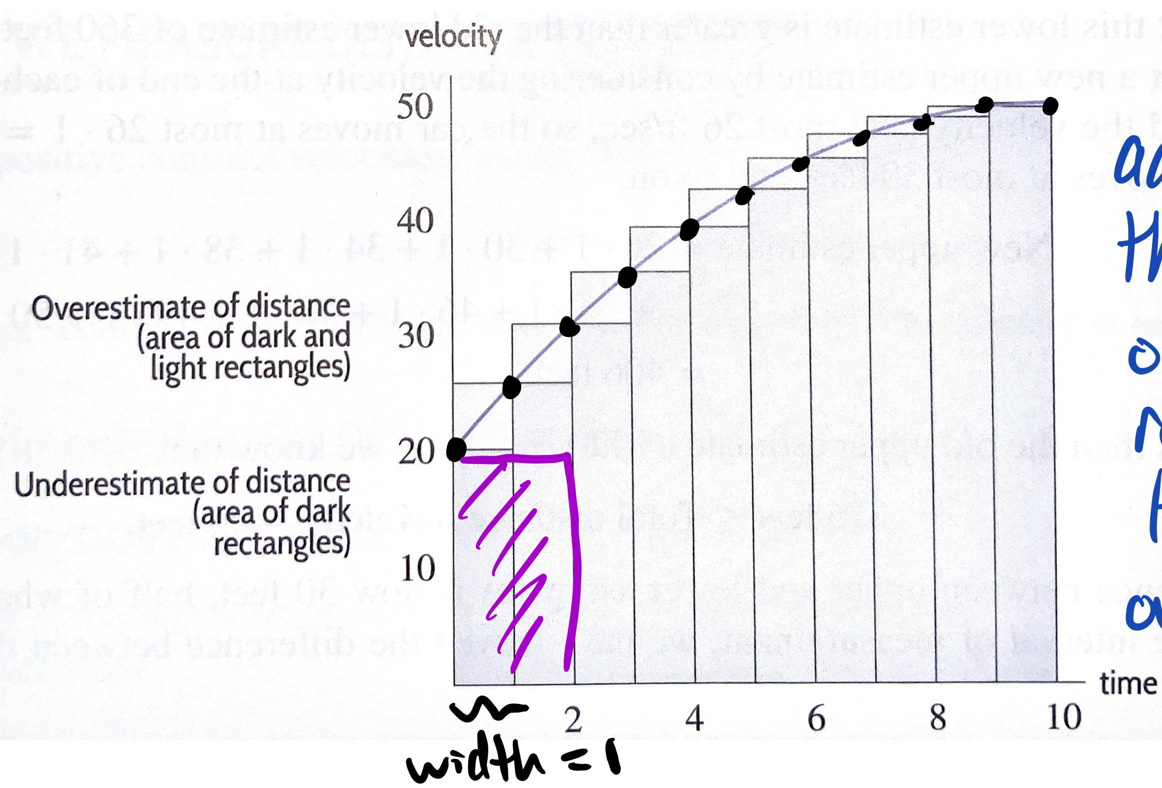
360

the underestimate is the sum of the areas of these rectangles overestimate



the overestimate is the sum of the areas of these rectangles

What if we had speed data from every 1 second instead of every 2 seconds?



add up the area of the rectangles to get over/under estimate

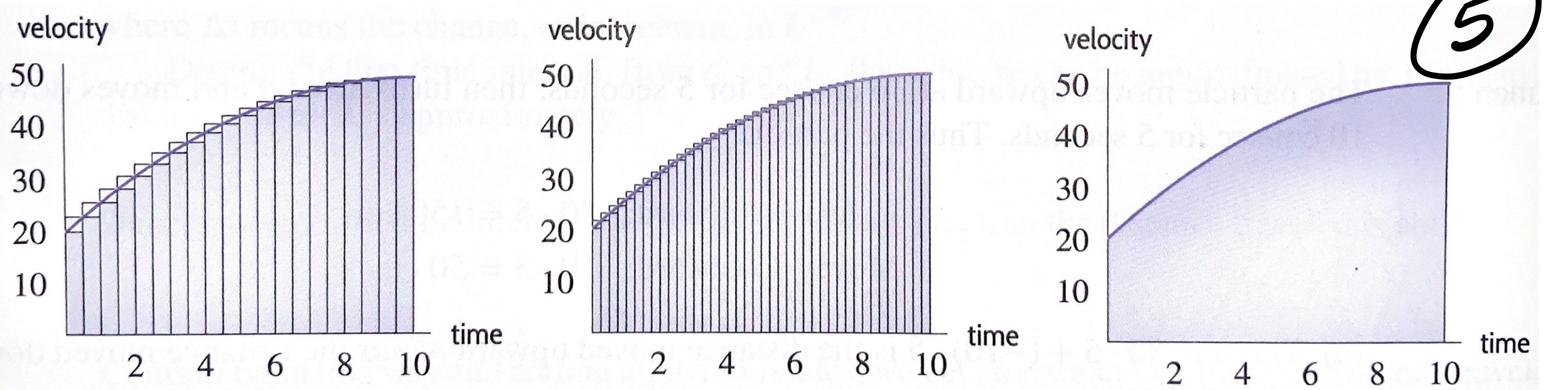


Figure 5.3: Velocity measured every 1/2 second

Figure 5.4: Velocity measured every 1/4 second

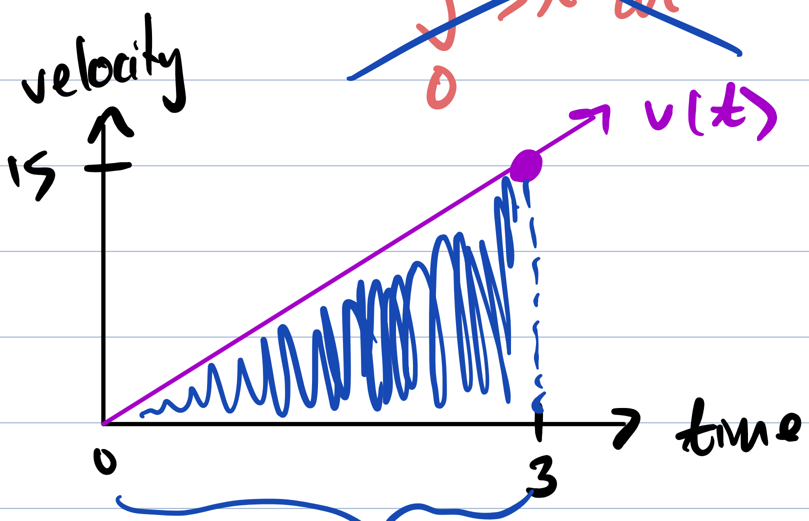
Figure 5.5: Distance traveled is area under curve

kind of a lie

The distance traveled is the area under the velocity curve.

<u>Example:</u> The velocity of a bicycle in feet per second is given by $v(t) = 5t$. How far does the bicycle travel in 3 seconds?

$$\cancel{\int_0^3 5t\,dt}$$

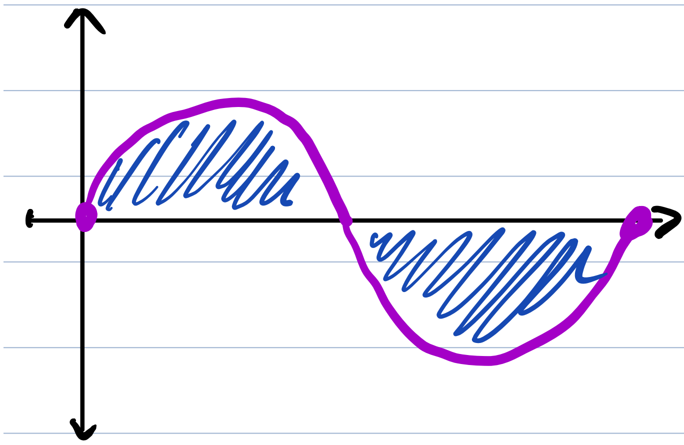


The answer is the area of the shaded region.

$$\text{Area} = \frac{1}{2} \cdot \text{base} \cdot \text{height} = \frac{1}{2} \cdot 3 \cdot 15 = \frac{45}{2} = \boxed{22.5 \text{ feet}}$$

Positive and Negative Velocity

We know that velocity measures more than speed, also direction.



velocity of a car accelerating forward, then braking then accelerating backward then braking

area above the x-axis counts as positive
area below the x-axis counts as negative

When you count area like this, then really the area under the curve represents "change in position from start to end"

If you really want total distance traveled then you count all the area as positive, whether above or below

Left and right sums

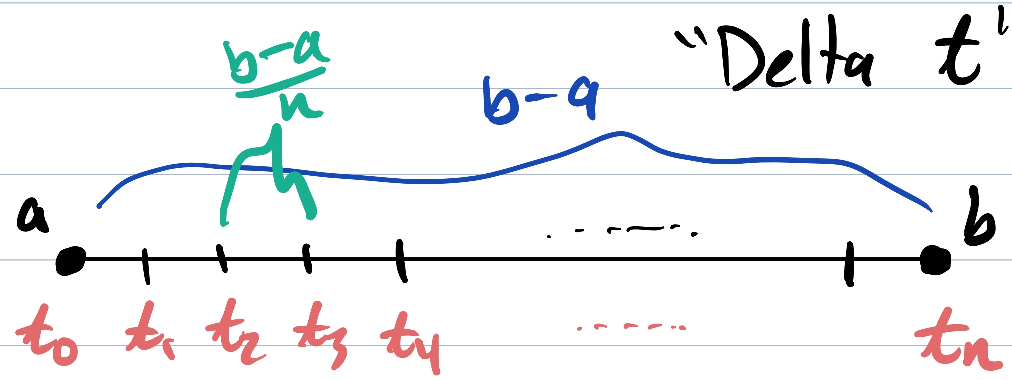
We can estimate area under the

curve by adding up the areas of a bunch of rectangles.

Suppose we start at $t=a$
we end at $t=b$

Suppose we want to estimate with n rectangles

Then the width of each rectangle is $\frac{b-a}{n} = \underbrace{\Delta t}$
"Delta t"



For the height of each rectangle we can use either the value of the function at the left end point OR the right endpoint

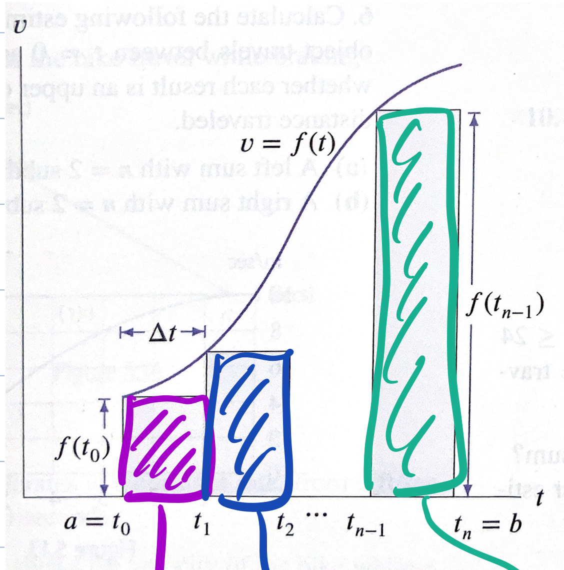


Figure 5.8: Left-hand sums

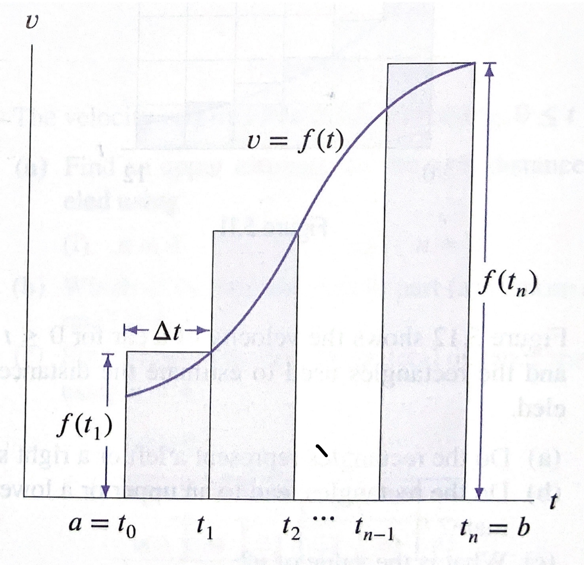


Figure 5.9: Right-hand sums

Formula for the left sum:

sum of area of rectangles

$$= \Delta t \cdot f(t_0) + \Delta t \cdot f(t_1) + \cdots + \Delta t \cdot f(t_{n-1})$$

$$= \Delta t \cdot \left(f(t_0) + f(t_1) + \cdots + f(t_{n-1}) \right)$$

Right sum:

$$= \Delta t \cdot \left(f(t_1) + f(t_2) + \cdots + f(t_n) \right)$$

0 Review of Ch. 17: Limits

$\lim_{x \to a} f(x)$ $\boxed{Ex\ 1}$ $\lim_{x \to 1} x^2 = (1)^2 = 1$

$$\lim_{x \to a} f(x) + g(x) = \lim_{x \to a} f(x) + \lim_{x \to a} g(x)$$

Different from Derivatives:

$$\lim_{x \to a} f(x) g(x) = \lim_{x \to a} f(x) \lim_{x \to a} g(x)$$ Product

$$\boxed{\lim_{x \to a} \frac{f(x)}{g(x)} = \frac{\lim_{x \to a} f(x)}{\lim_{x \to a} g(x)}}$$ Quotient

$\boxed{\text{Ex 2:}}$

$$\lim_{x \to 0} \frac{x^2 - x}{x - 1} = \frac{0^2 - 0}{0 - 1} = \frac{0}{-1} = 0$$

$\boxed{\text{Ex 3:}}$

$$\lim_{x \to 1} \frac{x^2 - x}{x - 1} = \frac{1^2 - 1}{1 - 1} = \frac{0}{0}$$

$$\lim_{x \to 1} \frac{x^2 - x}{x - 1} = \lim_{x \to 1} \frac{x\cancel{(x-1)}}{\cancel{x-1}} = \lim_{x \to 1} x = 1$$

$\boxed{\text{Ex 4}}$

$$\lim_{x \to 0} \frac{e^{2x} - 1}{x} = \frac{e^{2(0)} - 1}{0} = \frac{1-1}{0} = \frac{0}{0}$$