Monday, Nov 28 - Fall '22
Lecture #36

Announcements / Reminders

* Exam 3 on Wednesday in class (3.3-4.6)
* Wiley Plus #13 due Wed. night (4.6)
* Quiz 11 in discussion on Thursday (4.3, 4.6)

* ODS - Final exam scheduling deadline

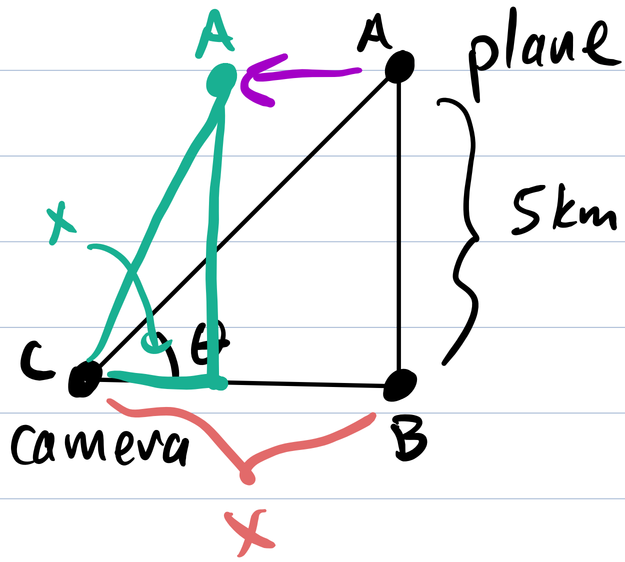
* Course Evaluations are open

Today: Finish 4.6, start 5.1.
Friday: Finish 5.1, go back and do 4.7.

* Final Exam is the Monday of Exam Week, 1pm-3pm

4.6 - Related Rates

An airplane, flying at 450 km/hr at a constant altitude of 5 km, is approaching a camera mounted on the ground. Let θ be the angle of elevation above the ground at which the camera is pointed. See Figure 4.93. When $\theta = \pi/3$, how fast does the camera have to rotate in order to keep the plane in view?



Two quantities
x and θ
they are related
How are $\frac{dx}{dt}$ and $\frac{d\theta}{dt}$
related?

Step 1) Write down a formula relating the two quantities.

$$\tan(\theta) = \frac{5}{x}$$

Step 2) Take the derivative of both sides with respect to a new variable t.

$$\tan(\theta(t)) = \frac{5}{x(t)}$$

$$\frac{d}{dt}\left[\tan(\theta(t))\right) = \frac{d}{dt}\left(\frac{5}{x(t)}\right)$$

$\frac{5}{t} = 5 \cdot t^{-1}$

$\hookrightarrow -5t^{-2}$

$$\frac{1}{\cos^2[\theta(t))}\underbrace{\theta'(t)} = -\frac{5}{(x(t))^2}\cdot\underbrace{x'(t)}$$

$$\underbrace{\frac{1}{\cos^2(\theta)} \cdot \frac{d\theta}{dt} = \frac{-5}{x^2} \cdot \frac{dx}{dt}}$$

this is an equation relating θ, x, $\frac{d\theta}{dt}$, $\frac{dx}{dt}$

If we know any 3, we can solve for the 4th.

Q: When θ is $\pi/3$, what is $d\theta/dt$?

Know: $\theta = \pi/3$

How can we find what x is when $\theta = \pi/3$?

$$\tan(\theta) = \frac{5}{x} \Rightarrow \tan\left(\frac{\pi}{3}\right) = \frac{5}{x}$$

$$\Rightarrow \sqrt{3} = \frac{5}{x}$$

$$\Rightarrow x = \frac{5}{\sqrt{3}} \text{ km}$$

Need $\frac{dx}{dt}$: -450 km/h

4

$$\frac{1}{\cos^2(\theta)} \cdot \frac{d\theta}{dt} = \frac{-5}{x^2} \cdot \frac{dx}{dt}$$

$$\Rightarrow \frac{1}{\cos^2(\pi/3)} \cdot \frac{d\theta}{dt} = \frac{-5}{\left(\frac{5}{\sqrt{3}}\right)^2} \cdot (-450)$$

$$\Rightarrow \frac{d\theta}{dt} = \frac{-5}{(25/3)} \cdot (-450) \cdot \cos^2(\pi/3)$$

$$\approx 67.5 \text{ rad/hr}$$

Does it make sense that θ is increasing?
Yes.

→ about 1 degree per second

Section 5.1 – How do we measure distance traveled?

Chapter 5: integrals

Suppose you're driving a car and as you're speeding up, you look down at the speedometer every 2 seconds and write down your speed.

time (sec)	0	2	4	6	8	10
speed (ft/sec)	20	30	38	44	48	50

Can you tell how far you traveled?

We don't know exactly because we don't have data for every millisecond, but we can estimate it!

* Between $t=0$ and $t=2$, you traveled at least $2 \cdot 20 = 40$ feet
* Between $t=2$ and $t=4$, you traveled at least $2 \cdot 30 = 60$ ft.

Overall: $\underbrace{2\cdot 20}_{0\to 2} + \underbrace{2\cdot 30}_{2\to 4} + \underbrace{2\cdot 38}_{4\to 6} + \underbrace{2\cdot 44}_{6\to 8} + \underbrace{2\cdot 48}_{8\to 10} + \cancel{2\cdot 50}$

$= 360$ feet

* must be an underestimate because you're always speeding up *

Overestimate: Use the right value of each 2 second window

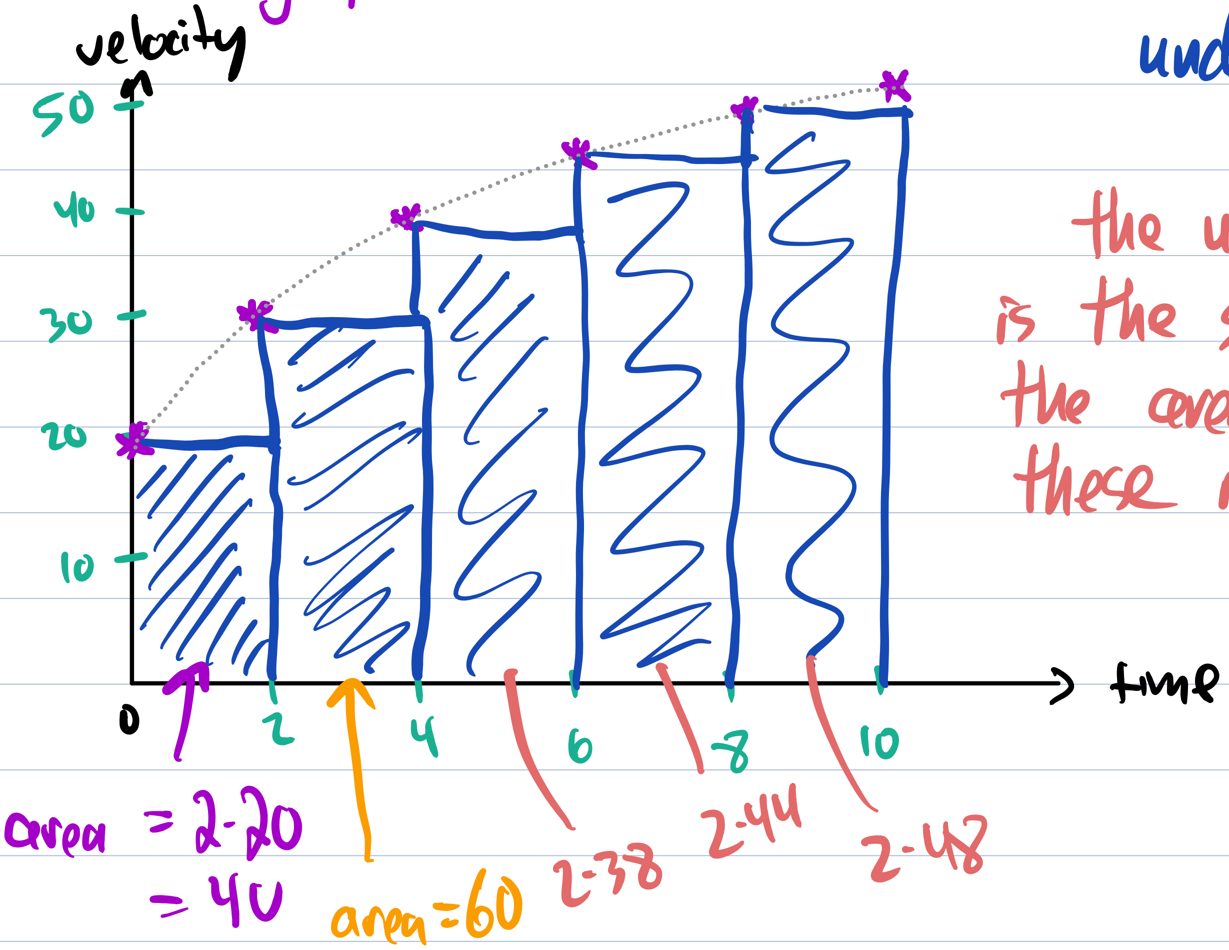
$$\begin{array}{ccccccccc} 0\to 2 & & 2\to 4 & & 4\to 6 & & 6\to 8 & & 8\to 10 \\ 2\cdot 30 & + & 2\cdot 38 & + & 2\cdot 44 & + & 2\cdot 48 & + & 2\cdot 50 \end{array}$$

$= 420$ feet

More accurate data (example: every 1 second) $\rightsquigarrow$ better estimates

time (sec)	0	2	4	6	8	10
speed (ft/sec)	20	30	38	44	48	50

As a graph:



underestimate

the underestimate is the sum of the areas of these rectangles