Wednesday, Nov. 16 - Fall '22
Lecture #33

Announcements / Reminders

* Wiley Plus #11 due tonight (3.10, 4.1, 4.2, ~~some 4.3~~)
* Quiz 10 tomorrow (same ↗)

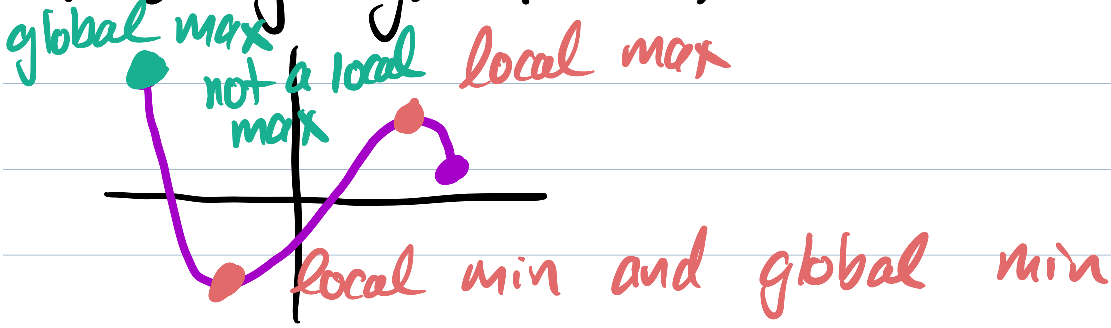
* Wiley Plus #12 assigned tomorrow morning and due TUESDAY night.

* Drop Deadline Fri.

* Next Monday - lecture
* Next Tuesday - discussion + office hours } + help desk
* Wed-Friday - no class, office hours, help desk

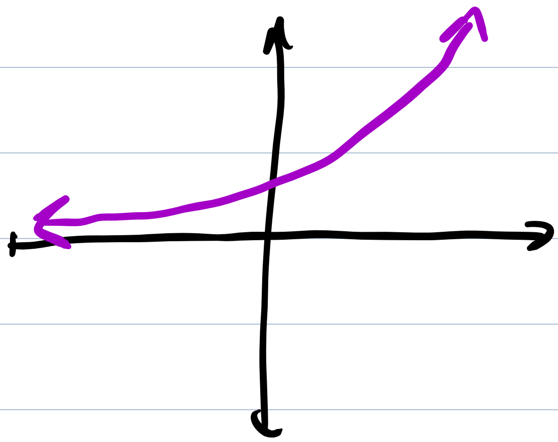
Section 4.2 - Optimization

Finding global maxima and minima.



Fact: If f is continuous on a closed interval (two endpoints, like $[a,b]$), then f <u>has</u> a global max <u>and</u> a global min.

Ex where this isn't true: $f(x) = e^x$ on $(-\infty, \infty)$

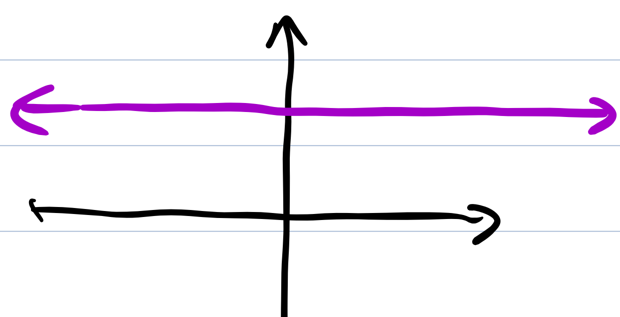


no global max
no global min

Ex: $f(x) = e^x$ on $[-5, 5]$

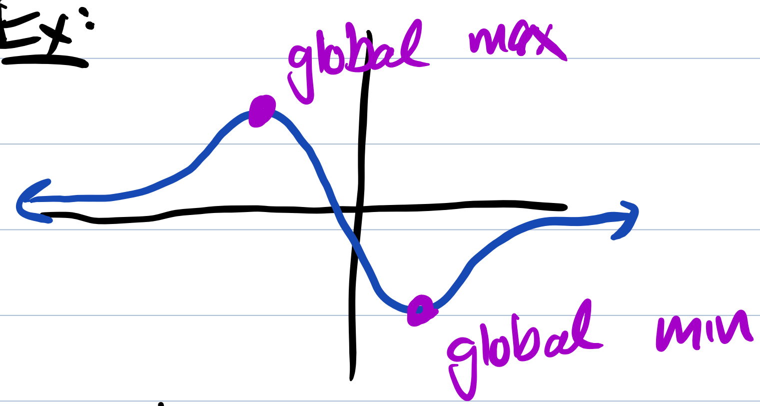
The fact guarantees that we have a global max and a global min

Ex: Horizontal Lines $f(x) = 3$



every point is a global max and a global min

Ex:



Strategy to find global max/min:

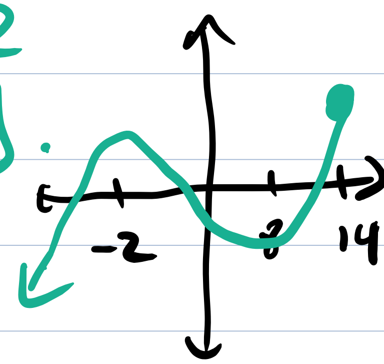
* Find the critical points

* Plug in both the critical points **and** the endpoints (if they exist) into f.

Almost true that: biggest of these = global max
smallest of these = global min

also have to consider the end behavior as $x \to \infty$ or $x \to -\infty$ if these are part of the domain.

Ex: Find the global extrema of
$f(x) = x^3 - 9x^2 - 48x + 52$
on the interval $(-\infty, 14]$.



critical points: -2, 8
endpoints: 14

$f(-2) = 104$
$f(8) = -396$
$f(14) = 360$

global max is $(14, 360)$ unless we go to $+\infty$ somewhere

~~global min is $(8, -396)$ unless we go to $-\infty$ somewhere~~

think about the shape of the graph

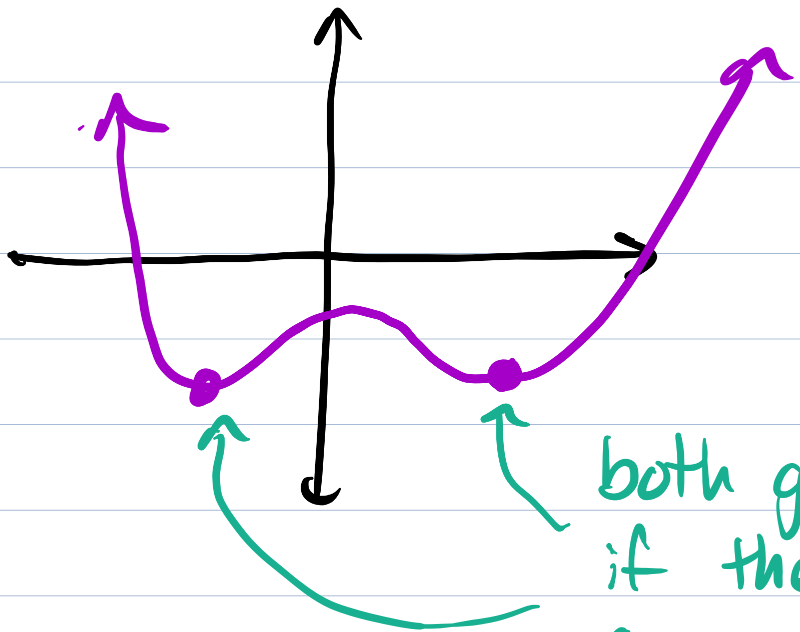
ours is cubic (degree 3 poly), leading coeff. positive $\Rightarrow$ the graph goes to $-\infty$ as $x \to -\infty$

So we don't really have a global min.

Final ans: global max: $(14, 360)$
no global min

Section 4.1 + 4.2 take lots of practice!

Ties are allowed



* Lecture video
* Exercises video } Fall 2020

Section 4.3 - Optimization and Modeling

This section: "Optimize some quantity, subject to some constraint."

find global max or min

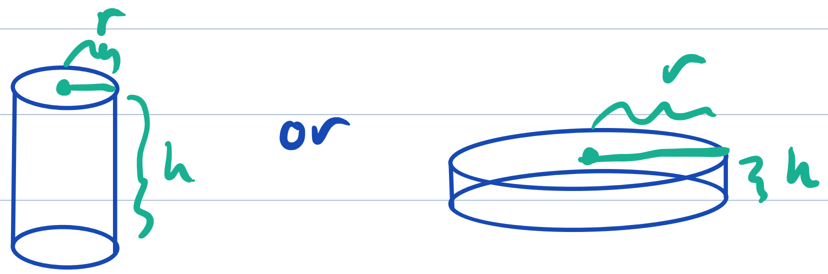
vol. of a box	fixed surface area
surface area of a box	fixed volume
area of a fenced yard	fixed amount of fence
gas consumption	fixed distance traveled

Example (from the book)

What are the dimensions of an aluminum can that holds 40 in^3 of juice that uses the least material.

Constraint: volume $= 40$

Goal: minimize surface area



you should know:
area of rect.
area of circle
area of a triangle
vol. of a rect. box
surface area. of a box

$$\text{Volume} = \pi r^2 h$$
$$\text{Surf. Ar} = 2\pi r^2 + 2\pi r h$$

Minimize $2\pi r^2 + 2\pi r h$ subject to $\pi r^2 h = 40$.

Step 1) Use the constraint to solve for one variable in terms of the other

Step 2) Plug that into the function you're optimizing

Step 3) Find the global extrema

$$\pi r^2 h = 40 \implies h = \frac{40}{\pi r^2}$$

$\hookrightarrow 2\pi r^2 + 2\pi r h \implies 2\pi r^2 + 2\pi r\left(\frac{40}{\pi r^2}\right)$

$$= 2\pi r^2 + \frac{80}{r}$$

Step 3: find the global minimum of

$r = \left(\frac{20}{\pi}\right)^{1/3} \approx 1.85$

$h = \sim \approx 3.7$

next class!