Monday, Nov. 14 - Fall '22
Lecture # 32

Announcements / Reminders

* Wiley Plus #11 due Wed (3.10, 4.1, 4.2, some ~~4.3~~)
* Quiz 10 Thurs. (same ↗)

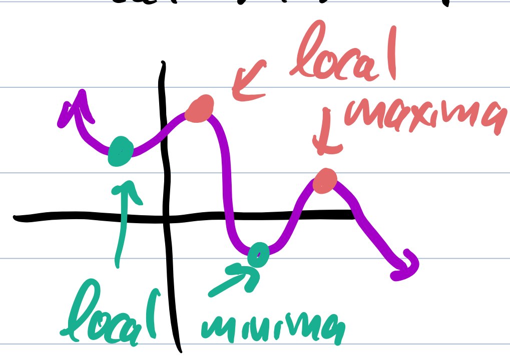
* Drop Deadline Fri.

* Next Monday - lecture
* Next Tuesday - discussion + office hours } + help desk
* Wed-Friday - no class, office hours, help desk

Section 4.1 - Using First and Second Derivatives

Goal: Find local minima and local maxima

Process: $f(x)$



1) Find the critical points.

$f'(x) = 0$ OR

$f'(x)$ DNE

these are the candidates for potential mins and maxes

2) Check each critical pt. to see if it's local max, local min, or neither

Method 1: First Derivative Test

Check whether f' is pos. or neg. a little to the left and right of the critical point.

$+ \to - \Rightarrow$ $\Rightarrow$ local max

$- \to + \Rightarrow$ $\Rightarrow$ local min

$+ \to +$

$- \to -$ or $\Rightarrow$ neither

Method 2: Second Derivative Test

Check the sign (pos. or neg.) of f'' at the critical point.

$f'' < 0 \Rightarrow$ $\Rightarrow$ local max

$f'' > 0 \Rightarrow$ $\Rightarrow$ local min

$f'' = 0$ or DNE $\Rightarrow$ no conclusion, could be max or min or neither

Ex: $f(x) = x^3 - 9x^2 - 48x + 52$

$f'(x) = 3x^2 - 18x - 48 = 3 \cdot (x-8)(x+2)$

$f''(x) = 6x - 18$

Critical Points: $p=-2,\quad p=8$

Second Deriv. Test:

$$f''(-2) = 6\cdot(-2)-18 = -12-18 = \text{neg}$$

$\Rightarrow$ local max at $x=-2$
$(-2, f(-2))$

$$f''(8) = 6\cdot 8 - 18 = \text{pos}$$

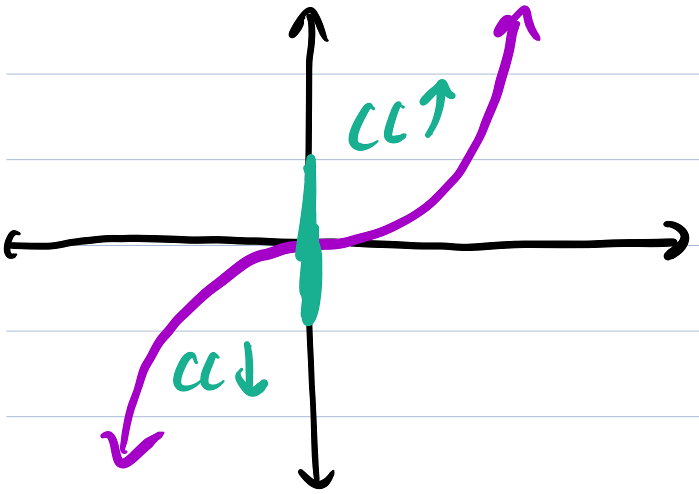
$\Rightarrow$ local min at $(8, f(8))$

Def: In addition to local extrema (where f' changes from $-$ to $+$ or from $+$ to $-$), we can also find where f'' changes from $-$ to $+$ or $+$ to $-$. These are called inflection points.

where f change concavity.

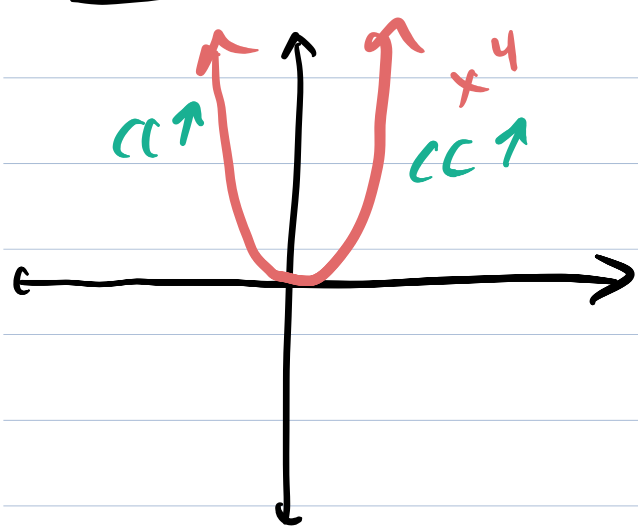
* If $f''(p)=0$ or $f''(p)$ DNE, then p is a possible inflection point.
* To check if p really is an inflection point, check a little to the left + right of p to see if f'' changes sign.

Ex: $f(x) = x^3$ $\qquad f''(x) = 6x$



inflection point at $(0,0)$

Ex: $f(x) = x^4$ $\qquad f''(x) = 12x^2$



possible infl. pt at $x=0$

$f''(-1) = 12 = \text{pos}$

$f''(1) = 12 = \text{pos}$

cc ↑ on both sides

not really an inflection point

Ex: Find the local extrema and infl. pts. of $g(x) = x \cdot e^{-x}$.

$g'(x) = (x)\cdot(-e^{-x}) + (1)(e^{-x})$

$\quad = e^{-x}(-x+1)$

$g''(x) = e^{-x}(x-2)$

(1) Find critical pts:

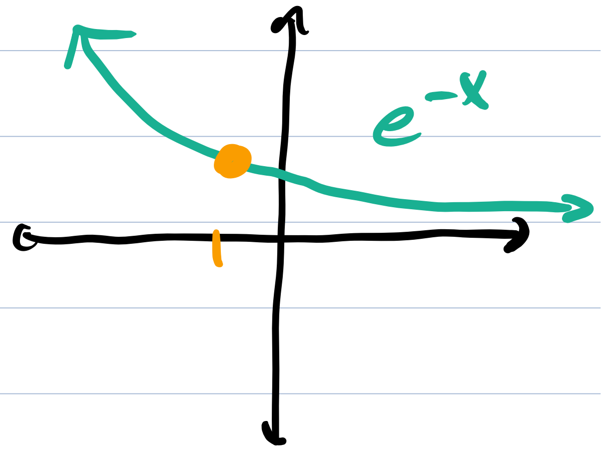
$g'(x) = 0$ or $g'(x)$ DNE

$$e^{-x}(-x+1)=0$$

$\Rightarrow e^{-x}=0$ OR $(-x+1)=0$

$\Rightarrow$ never $=0$ for any x-value

$\Rightarrow x=1$



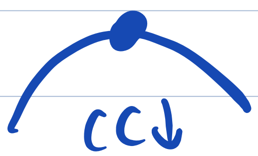
$g'(x)$ is never undefined

Only one critical point: $x=1$

(2) Is $x=1$ a max/min/neither?

SDT: $f''(1) = e^{-1}\cdot(1-2)$

pos • neg = neg



local max at $(1, g(1))$

$(1, 1\cdot e^{-1}) = (1, \frac{1}{e})$

Inflection points?

$$g''(x) = e^{-x}\cdot(x-2) = 0 \text{ or DNE}$$

$\Rightarrow x=2$

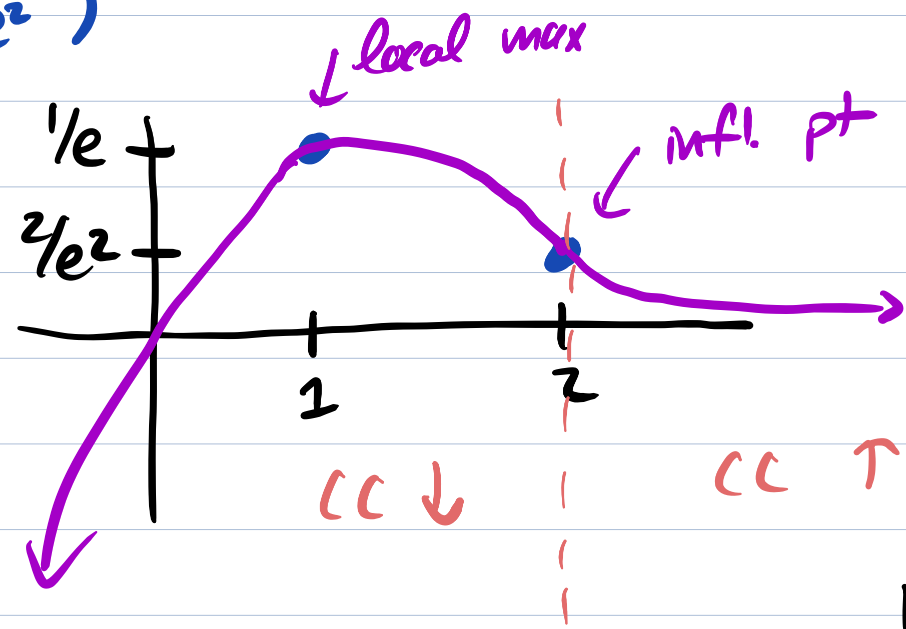
One possible inflection point.

$g''(1) = -e^{-1} < 0$ CC ↓

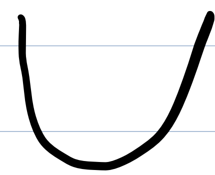
$g''(3) = e^{-3} > 0$ CC ↑

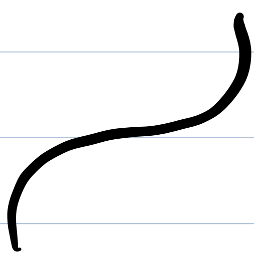
$(2, g(2))$ is really an inflection point
$= \left(2, \frac{2}{e^2}\right)$

Graph:



Analogy:





critical pts ⟷ possible inflection points

FDT or SDT ↓   plug in to f'' ↓

local max and min ⟷ inflection points

Section 4.2 – Optimization

Optimization means finding where in the domain a function is largest or smallest ever.

These are called global max and min.