Monday, Nov. 7 - Fall '22
Lecture #29

Skipping 3.7 + 3.8

Announcements / Reminders

* Wiley Plus #10 due Wed (3.4, 3.5, 3.6, 3.9)
* Quiz 9 Thursday (same)

* Exam 2 + key posted on website

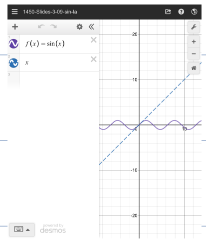
* If you are not sure if you should stay in class or withdraw, please email me and we'll talk!

* Grade Calculator (Desmos) on Course Website

Section 3.9 - Linear Approximations and the Derivative.

Ex: What is the linear approx. of $g(t) = \sin(t)$ near $t=0$?

$\sin(t) \approx t$ (near $t=0$)



②

Ex: What is the linear approx of $f(x) = e^{k \cdot x}$ near $x = 0$.

$\hookrightarrow e^{7 \cdot x}$

$a = 0$

Formula

$$f(x) \approx f(a) + f'(a) \cdot (x - a)$$

(for x-values close to a)

Secretly just the tangent line of f(x) at $x = a$.

$$f(0) = e^{k \cdot 0} = e^0 = 1$$

$$f'(x) = e^{k \cdot x} \cdot k = k e^{kx}$$

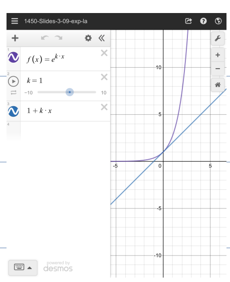
$$f'(0) = k \cdot e^{k \cdot 0} = k \cdot e^0 = k$$

$$f(x) \approx f(0) + f'(0) \cdot (x - 0)$$

$$= 1 + k \cdot x$$

$$= kx + 1$$

line with slope k and y-intercept 1



Are these approximations any good?

x	$\sin(x)$
0	0
0.01	0.0099998...
0.05	0.04997...
0.1	0.0998...
0.5	0.479...
1	0.84...

$k=3$

x	e^{3x}	$1+3x$
0	1	1
0.01	1.0304...	1.03
0.05	1.1618...	1.15
0.1	1.349...	1.3
0.5	4.48...	2.5
1	20.08...	4

Section 3.10 – Theorems about Differentiable Functions

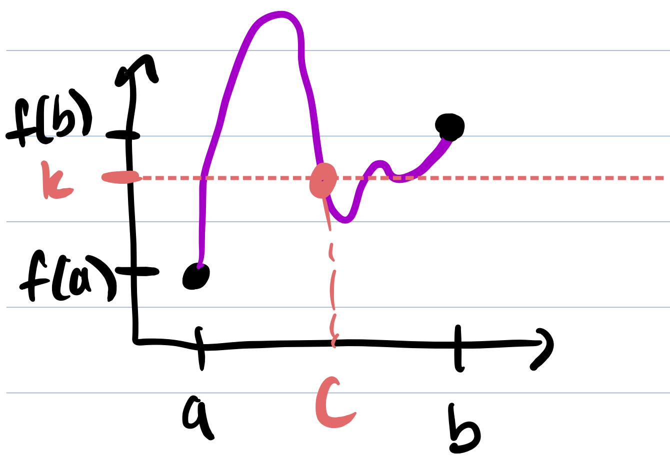
Intermediate Value Theorem (from Section 1.7)

Suppose $f(x)$ is continuous on the interval $[a,b]$. $a \le x \le b$

Then for any number k between $f(a)$ and $f(b)$, there's a point c in the interval $[a,b]$ with height k.

$(f(c)=k)$

Example: If your drink was 70° when you put it in the fridge and 30° when you took it out, then it was exactly 40° at some point in between.



this continuous on the interval [a,b]

there exists a "c" between a and b where the height of the function is k

The Mean Value Theorem (MVT)

If f is continuous and differentiable on an interval [a, b], then there exists some number c between a and b such that:

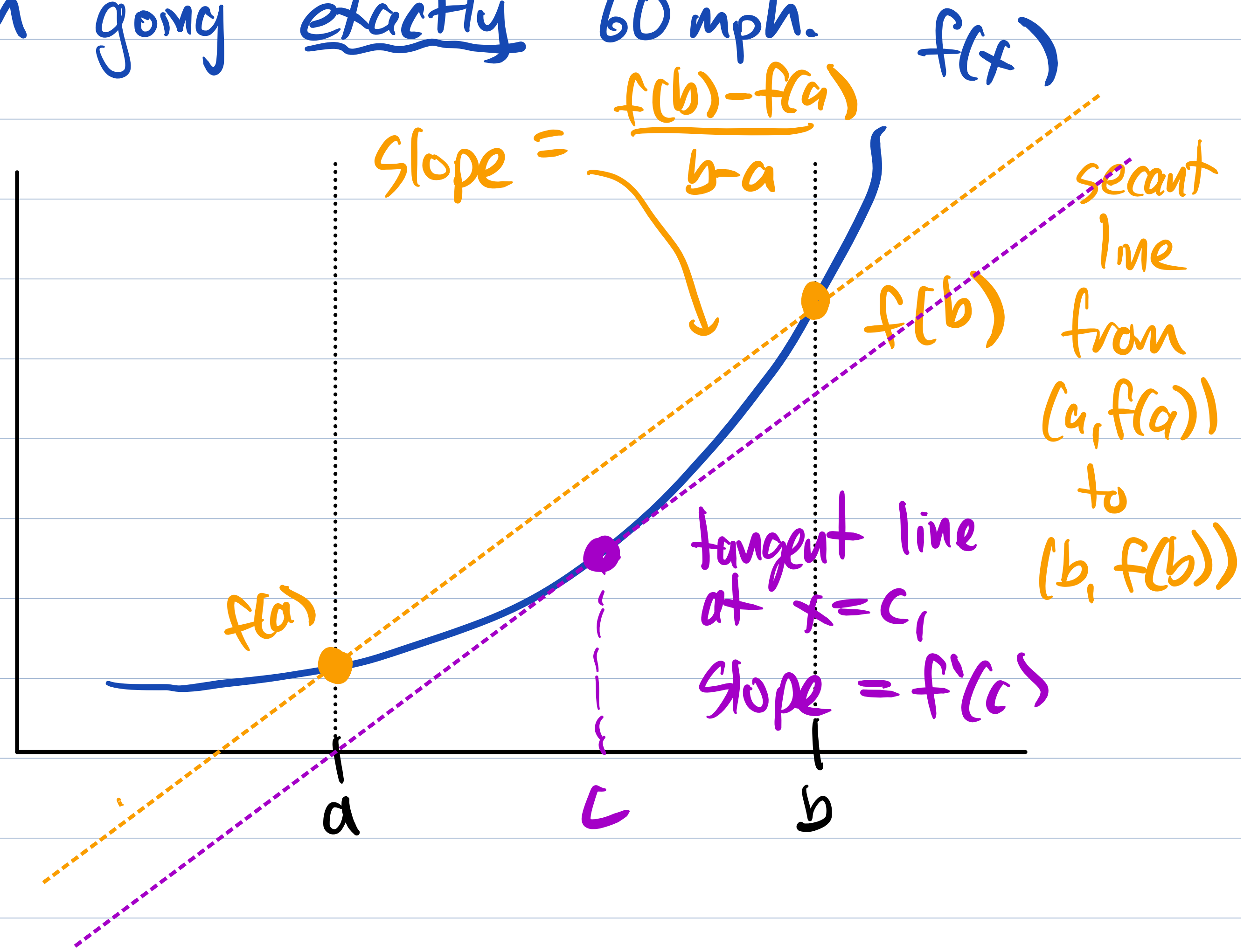
$$f'(c) = \underbrace{\frac{f(b)-f(a)}{b-a}}$$

instantaneous velocity at x=c (slope of TL)

average velocity over the interval (slope of the secant line)

Translation: At some point in a journey, your instantaneous velocity will be the same as your average velocity over the entire journey.

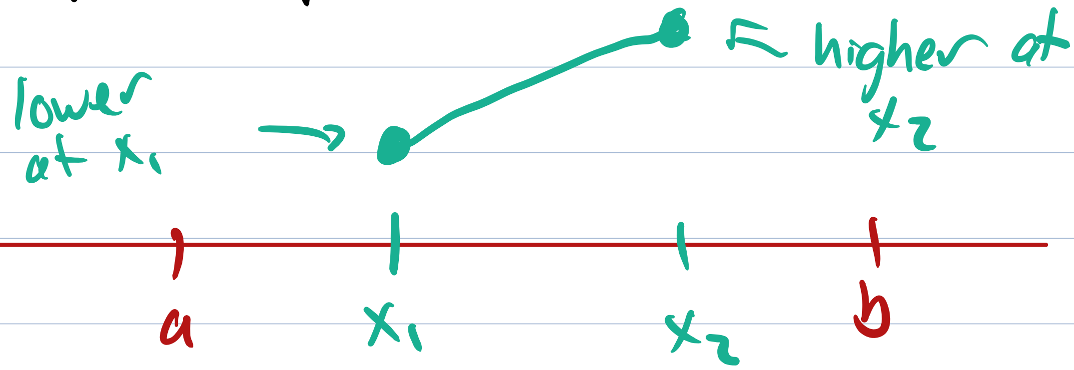
Ex: If you drive 180 miles in 3 hours, then at some point you must have been going exactly 60 mph.



Increasing Function Theorem

Def: A function $f(x)$ is increasing on the interval $[a,b]$ if:

for all x_1 and x_2 in the interval, if $x_1 < x_2$, then $f(x_1) < f(x_2)$.



Increasing Function Theorem:

If $f(x)$ is cns. and diff'ble on the interval $[a,b]$ and $f'(x) > 0$ at all x-values in the interval, then, $f(x)$ is increasing in $[a,b]$.

The Constant Function Theorem:

If $f(x)$ is cns. and diff'ble on the interval $[a,b]$ and $f'(x) = 0$ at all x-values in the interval, then: $f(x)$ is a constant (horizontal line) everywhere in $[a,b]$.

$f'(x) = 0 \Rightarrow f(x)$ is constant

Next time: The Racetrack Principle