Wednesday, Nov. 2 - Fall '22
Lecture #27

Announcements / Reminders

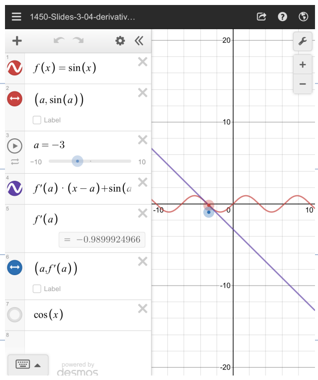
* Wiley Plus #9 today (3.2, 3.3, some 3.4)
* Quiz 8 Thursday (same ↗) sugg HW from 3.2, 3.3, 3.4

* Exam Stuff
  - average + distribution about the same as last time
  - I have not yet adjusted E1 scores for those who improved on E2
  - Exams are not individually curved, but the course as a whole is depending on the final grade distribution.
    Very rough estimate: add 6 pts (out of 60) to your E1 and E2 scores (not a guarantee in any way!)
  - Some of you will need to start thinking about whether withdrawing is in your best interest. Discuss with your advisors and I can help you talk through it as well.

Drop deadline: Nov. 18 (Friday)

Section 3.5: The Trigonometric Functions

Goal: Figure out the deriv. of sine, cosine and tangent.



WARNING: Everything in the future about trig functions is in radians, not degrees.

Fact #1: $\frac{d}{dx}[\sin(x)] = \cos(x)$

Fact #2: $\frac{d}{dx}(\cos(x)) = -\sin(x)$

Ex: What is the second derivative of $\sin(x)$?

Let $f(x) = \sin(x)$. By fact #1, $f'(x) = \cos(x)$. $f''(x)$ is the derivative of $\cos(x)$, which is $-\sin(x)$.

$$f'''(x) = -\cos(x)$$

$$f^{(4)}(x) = -(-\sin(x)) = \sin(x)$$

Ex: $\frac{d}{d\theta}\left(2\cdot\sin(3\theta)\right)$

$f(\theta) = \sin(\theta)$
$g(\theta) = 3\theta$
$f'(\theta) = \cos(\theta)$
$g'(\theta) = 3$

$$= 2\cdot\frac{d}{d\theta}\left(\sin(3\theta)\right)$$

$$= 2\cdot\left(f'(g(\theta))\cdot g'(\theta)\right)$$

$$= 2\cdot\left(f'(3\theta)\cdot 3\right)$$

$$= 2\cdot\cos(3\theta)\cdot 3 = 6\cos(3\theta)$$

Section 3.1: $\frac{d}{dx}\left(c\cdot f(x)\right)$

$$= c\cdot f'(x)$$

Ex: $\frac{d}{dx}\left(5x^3\right)$

$$= 5\cdot\frac{d}{dx}\left(x^3\right) = 5\cdot\left(3x^2\right) = 15x^2$$

Ex: $\frac{d}{dx}\left(\cos^2(x)\right)$

$\cos^2(x)$ means $(\cos(x))^2$

$= \frac{d}{dx}\left((\cos(x))^2\right)$

inside $= g(x) = \cos(x)$ $\quad g'(x) = -\sin(x)$

outside $= f(x) = x^2$ $\quad f'(x) = 2x$

$= f'(\cos(x)) \cdot (-\sin(x))$

$= 2\cdot\cos(x)\cdot(-\sin(x)) = -2\cos(x)\sin(x)$

inside

$\frac{d}{dx}\left(f(g(x))\right) = f'(g(x))\cdot g'(x)$

outside

Unrelated: Wiley Plus

$2ex+1$

2*e*x +1

Ex: $\frac{d}{dx}\left(\cos(x^2)\right)$

outside $= f(x) = \cos(x)$
inside $= g(x) = x^2$

$f'(x) = -\sin(x)$
$g'(x) = 2x$

$= f'(g(x)) \cdot g'(x)$

$= f'(x^2) \cdot (2x)$

$= -\sin(x^2) \cdot 2x = -2x\sin(x^2)$

Tangent: What is the derivative of $\tan(x)$?

We can figure this out. $\tan(x) = \frac{\sin(x)}{\cos(x)}$

$$\frac{d}{dx}(\tan(x)) = \frac{d}{dx}\left(\frac{\sin(x)}{\cos(x)}\right) = \frac{g(x)f'(x) - f(x)g'(x)}{(g(x))^2}$$

$$= \frac{\cos(x)\cdot\cos(x) - \sin(x)\cdot(-\sin(x))}{\cos^2(x)}$$

$$= \frac{\cos^2(x) + \sin^2(x)}{\cos^2(x)} = \frac{1}{\cos^2(x)}$$

$f(x) = \sin(x)$ $\quad f'(x) = \cos(x)$

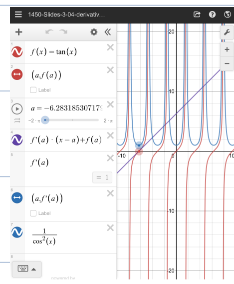
$g(x) = \cos(x)$ $\quad g'(x) = -\sin(x)$

<u>Fact #3:</u> $\frac{d}{dx}\left(\tan(x)\right) = \boxed{\frac{1}{\cos^2(x)}} = \sec^2(x)$

→ vert asymps whenever $\cos(x)$ is 0

$x = \frac{\pi}{2}, \frac{3\pi}{2}, \ldots$

→ never negative



<u>Ex:</u> $\frac{d}{d\theta}\left(\tan(1-\theta)\right)$

outside $= f(\theta) = \tan(\theta)$

inside $= g(\theta) = 1-\theta$

$f'(\theta) = \frac{1}{\cos^2(\theta)}$

$g'(\theta) = -1$

$f'(g(\theta)) \cdot g'(\theta)$

$= f'(1-\theta) \cdot (-1)$

$= \frac{1}{\cos^2(1-\theta)} \cdot (-1) = -\frac{1}{\cos^2(1-\theta)}$