Monday, Oct. 17 – Fall '22
Lecture #21 / 41

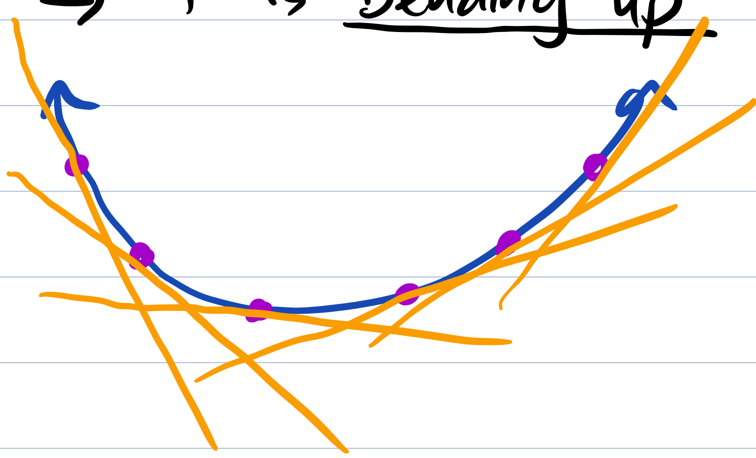
Announcements / Reminders

* Wiley Plus #7 due Wed (2.3, 2.4, 2.5)
* No discussion Thurs, no lecture Fri!
  (so no quiz this week)
* Exam 2 is Wed, Oct 26
  covers material from Fri, Sept 30
  to Monday, Oct 24
* Help Desk reminder!

Section 2.5 – The Second Derivative

$f' > 0 \Rightarrow$ f is going up (incr.)
$f' < 0 \Rightarrow$ f is going down (decr.)

$f'' > 0 \Rightarrow$ f is bending up (cc ↑)



$f'' < 0 \Rightarrow f$ is <u>bending down</u> (cc↓) ②

$f'' > 0$ or $f'' < 0$?

Example 1 For the functions graphed in Figure 2.46, what can be said about the sign of the second derivative?

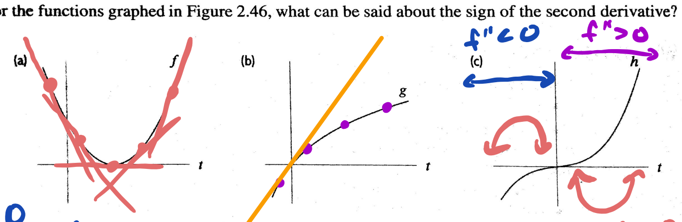


Figure 2.46: What signs do the second derivatives have?

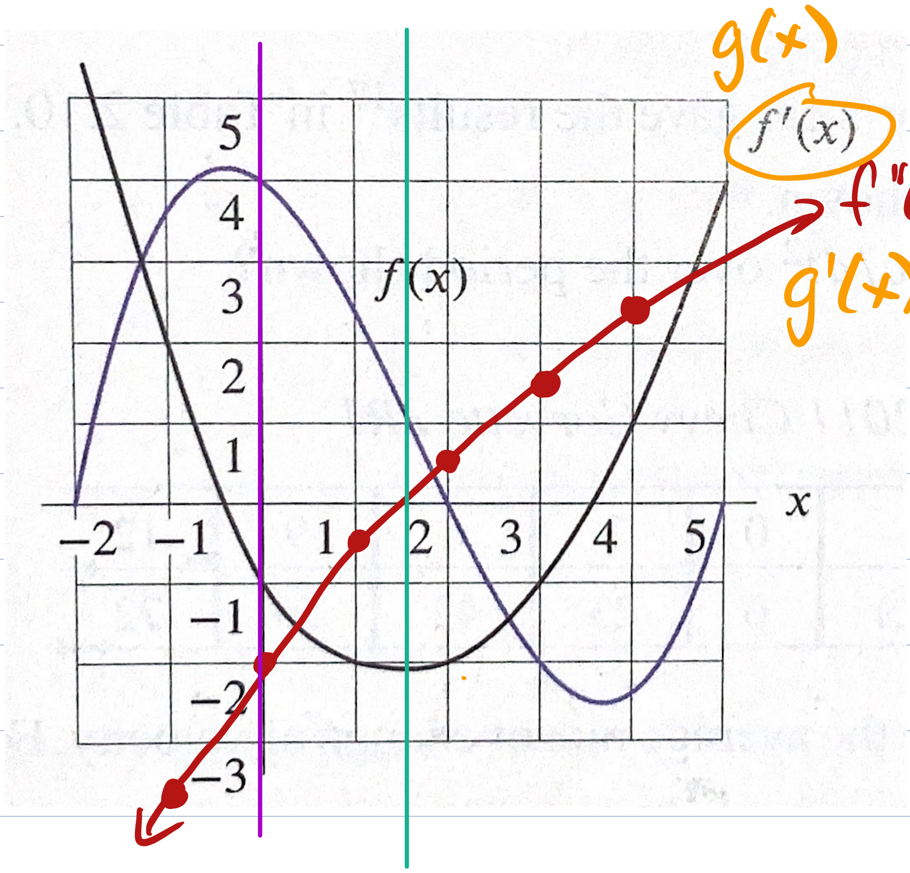
$f'' = 0$ at $x = 0$

$f'' > 0$ everywhere
cc↑

$f'' < 0$ everywhere
cc↓
$f' > 0$ everywhere

$f' = 0$ at $x=0$
$f' > 0$ everywhere else

Ex: Draw $f''(x)$.

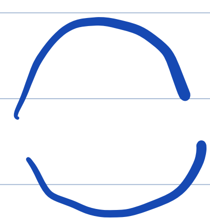


$f''(x)$ is the derivative of $f'(x)$

$f''(-1) \approx -3.5$

$f''(0) \approx -2$

$f''(1) \approx -\frac{1}{2}$



$f'' < 0$

$f'' > 0$

At $x=0$: $f>0$ - above ground
$f'<0$ - going downward
$f''<0$ - bending downward

Rates of Change:

First derivative: rate of change of $f(x)$

→ The roller coaster is going up:
$f'>0$

→ The economy is shrinking
$g'<0$

Second derivative: rate of change of the rate of change of $f(x)$

→ The steepness of the roller coaster is increasing
$f''>0$

→ The rate at which the economy is shrinking is decreasing:
$g''>0$

Position – Velocity – Acceleration

Position: $s(t)$

Velocity: $v(t) = s'(t)$

Acceleration: $a(t) = v'(t)$
$= s''(t)$

Section 2.6 – Differentiability

We say that a function $f(x)$ is differentiable at $x=a$ if:

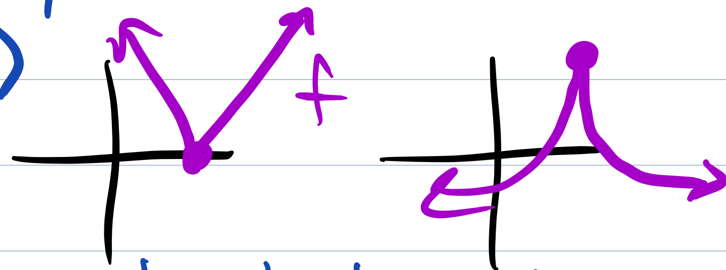
* the derivative exists at $x=a$

(or, rephrased)

* $\displaystyle\lim_{h\to 0} \frac{f(a+h)-f(a)}{h}$ exists

What could make a function NOT differentiable? In other words, what could make $\displaystyle\lim_{h\to 0} \frac{f(a+h)-f(a)}{h}$ not exist?

1) If $f(a)$ doesn't exist
2) If f is not continuous at $x=a$
(we'll justify this later)
3) f has a sharp corner at $x=a$
(not smooth)



4) f has a vertical tangent line at $x=a$

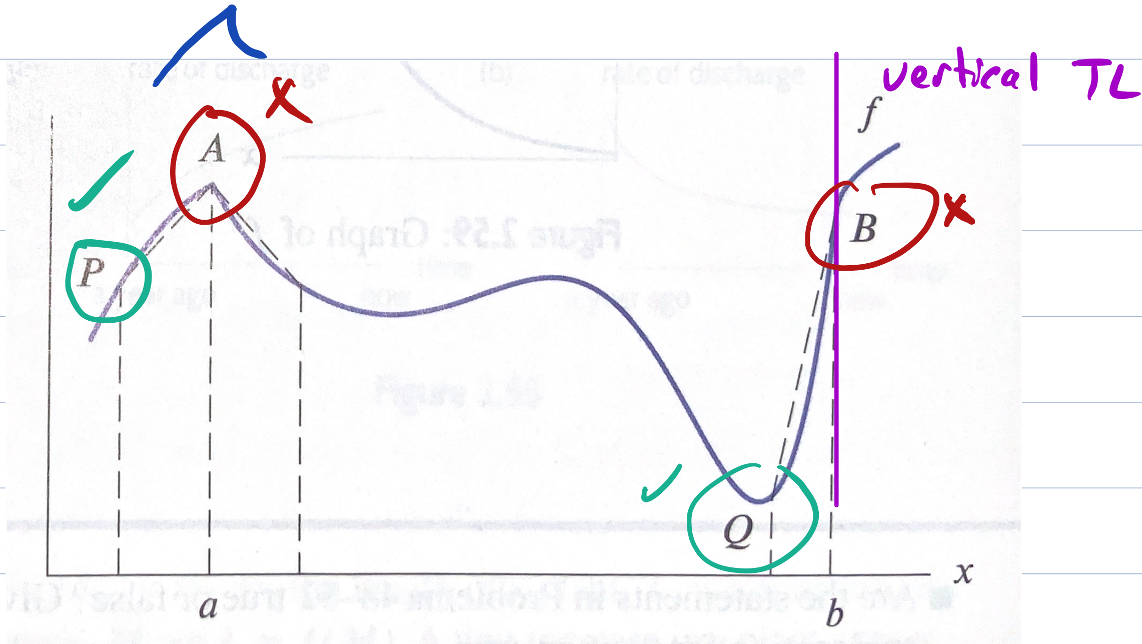
$f(x) = \sqrt[3]{x}$

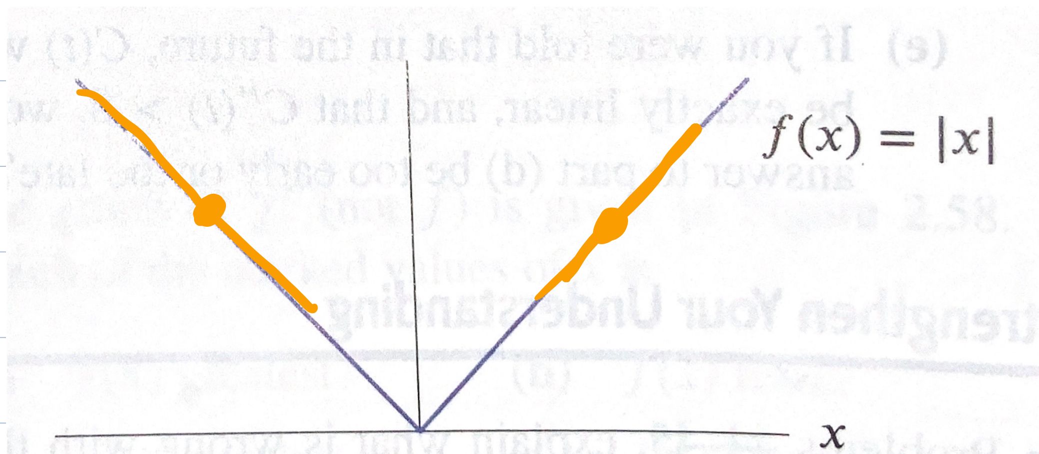
you're not allowed to say $f'(0) = \infty$.

So, this function is <u>not</u> diff'ble at $x=0$, but <u>is</u> everywhere else.

What are the intervals on which $f(x) = \sqrt[3]{x}$ are diff'ble?
$(-\infty, 0)$ $(0, \infty)$

Ex: Where is the function NOT diff'ble? ⑥

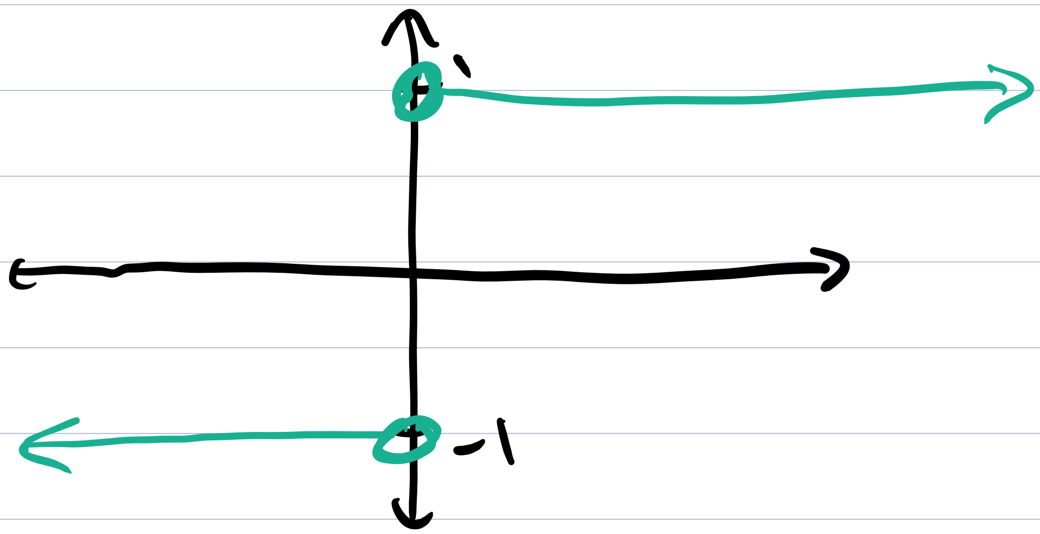




What is $f'(x)$?

Easier: look at the graph.

$$f'(x) = \begin{cases} -1, & x<0 \\ \text{DNE}, & x=0 \\ 1, & x>0 \end{cases}$$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