Wednesday, Oct 12 – Fall '22
Lecture #19

Announcements / Reminders

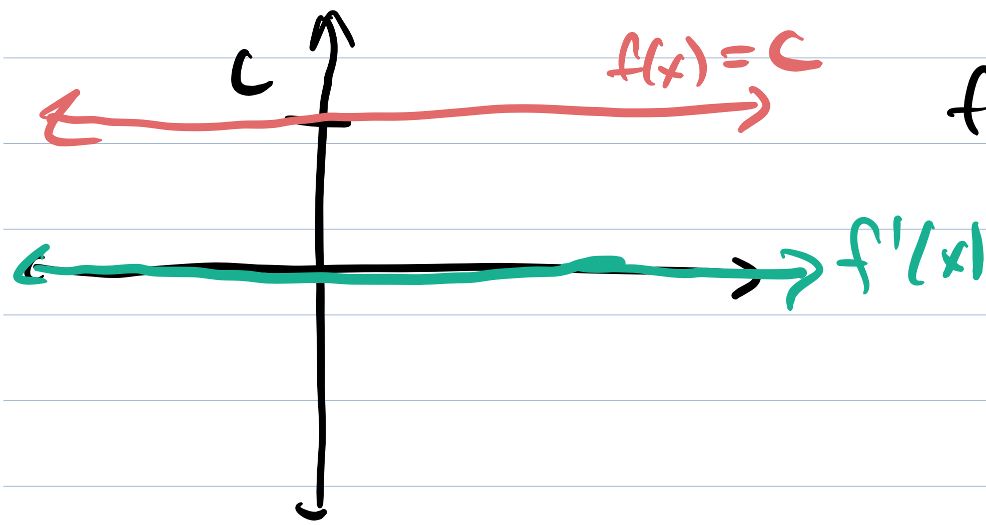
* Wiley Plus #6 due tonight (2.1, 2.2)
* Quiz #6 Thurs (2.1, 2.2)
* Exam 2 is Wed, Oct 26 (not next week)

First Derivative Formulas: (2.3)

Let $f(x)$ be a constant function.

$$f(x) = C \qquad (f(x) = 5)$$

for some # C.

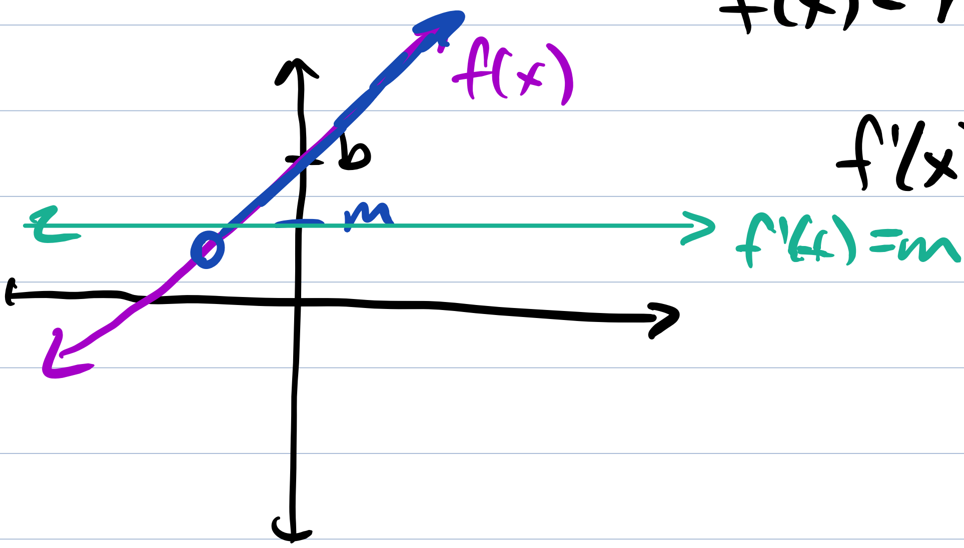


What is $f'(x)$?

$f'(x) = 0$ for all x.

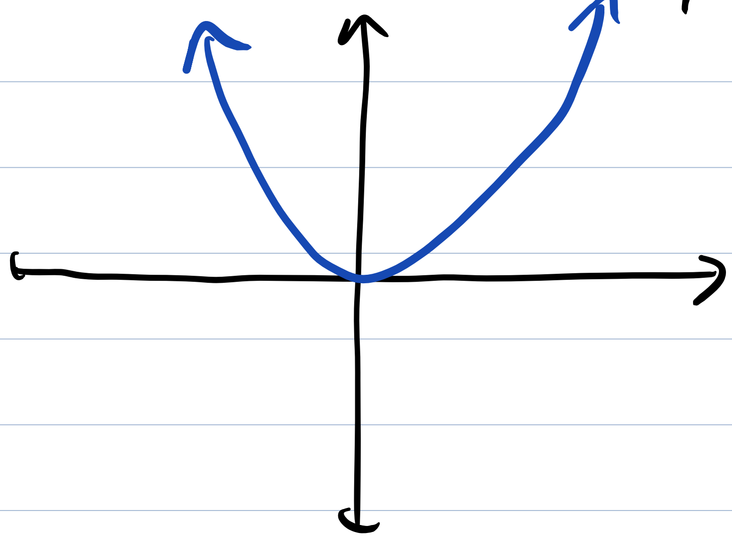
Let $f(x)$ be a line:

$$f(x) = m \cdot x + b$$



$$f'(x) = m$$

<u>Ex:</u> If $f(x) = x^2$, what is $f'(x)$?



$$f(x) = x^2$$

Limit definition of derivative:

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{(x+h)^2 - x^2}{h}$$

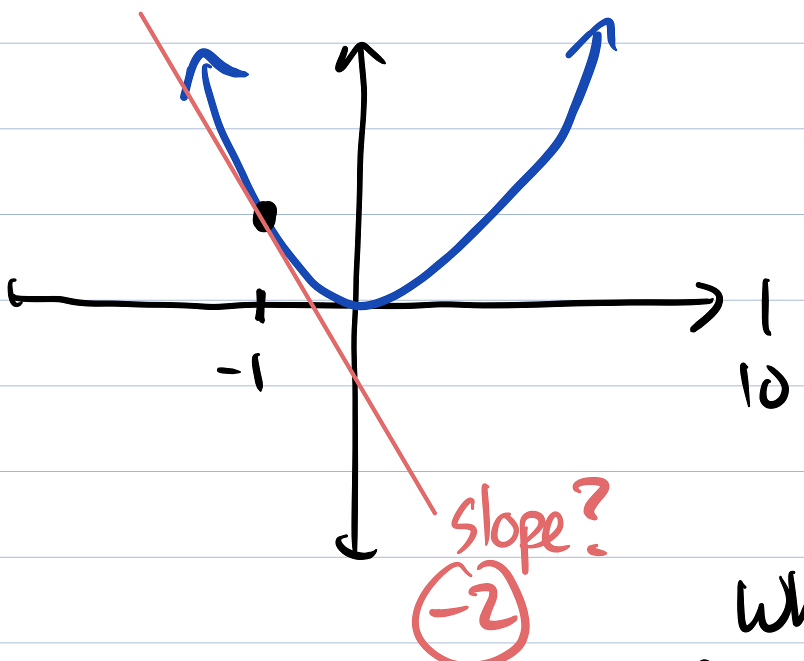
③

$$= \lim_{h\to 0} \frac{(x^2+2xh+h^2)-x^2}{h}$$

$$= \lim_{h\to 0} \frac{2xh+h^2}{h} = \lim_{h\to 0} \frac{\cancel{h}\cdot(2x+h)}{\cancel{h}}$$

$$= \lim_{h\to 0} (2x+h) = 2x$$

The derivative of $f(x)=x^2$ is $f'(x)=2x$.



What is the slope of the T.L. of x^2 at $x=-1$.

$f'(-1) = -2$

What is the slope of the T.L. at $x=10$?

$f'(10) = 20$ ✓

Sneak Peek: Der. of x^2 is $2x$
Der. of x^3 is $3x^2$

Der. of x^4 is $4x^3$
Der. of x^5 is $5x^4$

Power Rule: If $f(x) = x^n$, then
$f'(x) = n \cdot x^{n-1}$

Actually works even when n is not a whole #:
Der. of $x^{5/3}$ is $\frac{5}{3}x^{2/3}$.

Section 2.4: Interpretations of the Derivative

Calculus was invented in the 1600s nearly simultaneously by two people:
Newton, Leibniz

They had different viewpoints. We teach Newton's version but with Leibniz's notation.

Notations for a derivative:

Newton (physics) – $\dot{y}$ or $\frac{\dot{y}}{\dot{x}}$ (1704)

Lagrange – $f'(x)$ (1770)

Euler "Oiler" – $(Df)(x)$ (1700s)

Leibniz – $\frac{dy}{dx}$ (1680s)

Pretty useful

$y = x^2$
$f(x) = x^2$

Let $y = f(x)$ be a function.

Leibniz's

Lagrange

$$\frac{dy}{dx} = \underbrace{f'(x)}_{\text{slope of the TL of } f(x) \text{ at a certain } x\text{-value}}$$

ratio

slope of the TL of f(x) at a certain x-value

$$\frac{\text{small } y \text{ change}}{\text{small } x \text{ change}}$$

= when x changes a little, how much does y change?

$f(10) = 5$
$f'(10) = 2$
Estimate $f(11)$

Ex: $\frac{dy}{dx} = 2$ means if x changes a little, then y changes by twice as much

$\rightarrow 0.1$ $\quad \uparrow 0.2$

Another use of this notation is to use " $\frac{d}{dx}$ " as a command to "take the derivative".

$$\underbrace{\frac{d}{dx}}_{\text{"The derivative of" with respect to x}} \underbrace{\left(x^2\right)}_{x^2} \underbrace{=}_{\text{is}} \underbrace{2x}_{2x}$$

$\cancel{x^2 = 2x}$

$$\frac{d}{dz}(z^2) = 2z$$

Der. of x^5 is $5x^4$

$$\Rightarrow \quad \frac{d}{dx}(x^5) = 5x^4$$

Let $y = x^5$. $\quad \frac{dy}{dx} = 5x^4$

$$\frac{d}{dx}(y) = 5x^4 \qquad \frac{d}{dx}(x^5) = 5x^4$$

$$y' = 5x^4$$

Leibniz notation makes units make sense.

Suppose $s = f(t)$ measures the position of a car, in meters, after t seconds.

$$\frac{ds}{dt} \quad \begin{matrix} \leftarrow \text{meters} \\ \leftarrow \text{seconds} \end{matrix} \quad \Big\} \quad \frac{\text{meters}}{\text{sec}} \qquad \text{"meter per second"}$$

Unit of $\underset{\substack{\| \\ 100}}{f(3)}$: meters

Unit of $\underset{-5}{f'(3)}$: meters/sec

<u>Ex:</u> The cost of extracting T tons of ore from a copper mine is $C = f(T)$ dollars. What does it mean to say $f'(2000) = 100$?

Lagrange $\quad\quad\quad$ Leibniz

$$f'(2000) = \boxed{\frac{dC}{dT} = 100} \quad \frac{\$}{\text{ton}}$$

dollars per ton

pretend this a fraction

"$dC = 100 \cdot dT$"

"$\Delta C = 100 \cdot \Delta T$"

"When 2000 tons of ore have already been extracted, the additional cost to extract the next little amount is about 100 dollars/ton."