Wed, Oct. 5 - Fall '22
Lecture #16

Announcements / Reminders

* I'm so sorry exam grades weren't posted after class on Monday!

* Wiley Plus #5 due tonight (1.8 and 1.9)
* Quiz #4 tomorrow (1.8, 1.9, 2.1)

How to succeed in Calculus

or

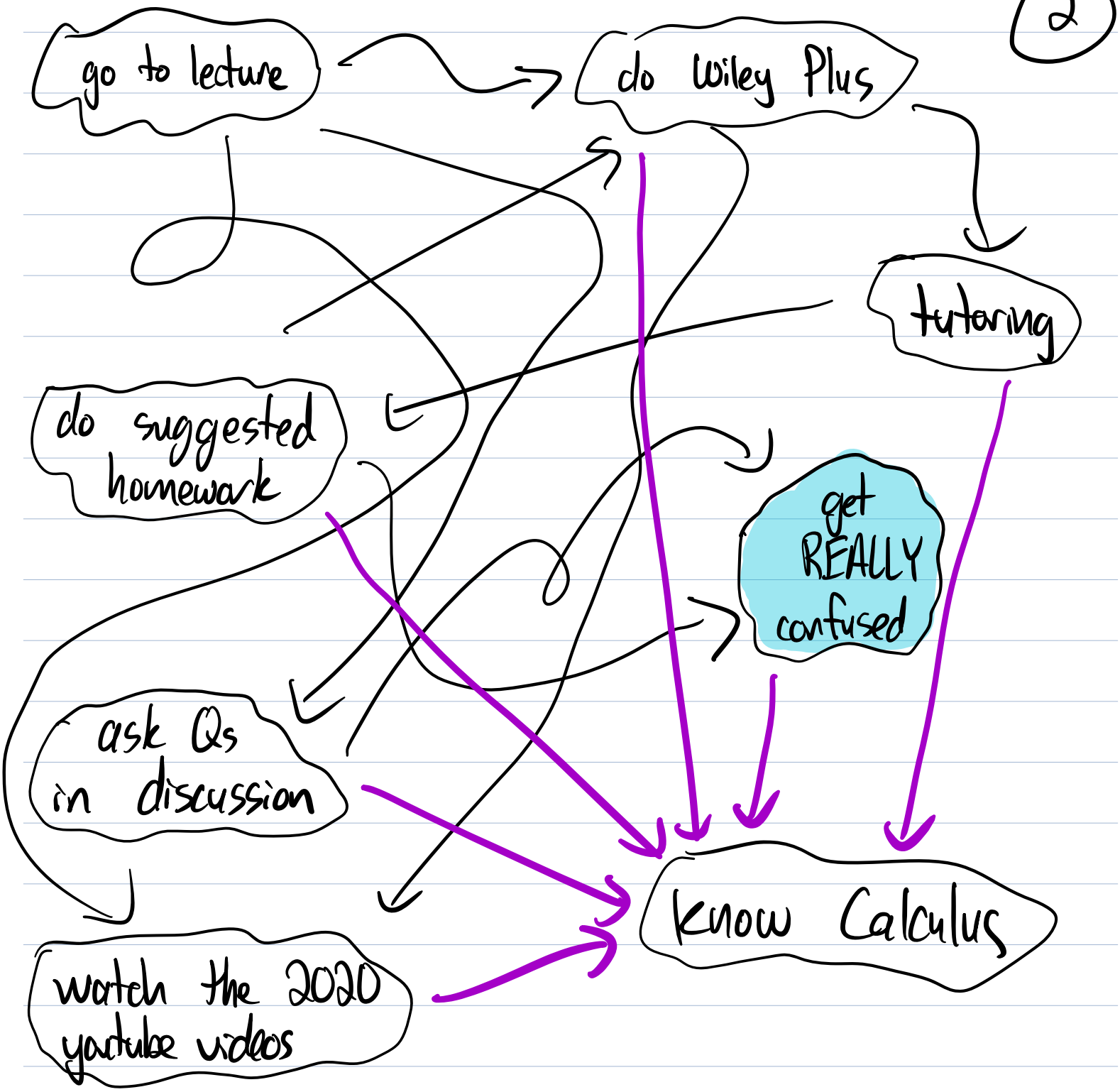
"Get an A in Calc with these 5 easy tricks. Professors hate #3!"

(1) Understand the learning process.

It's not:

go to lecture ⟶ know Calculus

It's more like:



(2) Work on Wiley Plus as we go through the lectures, not all in the last few days/hours.

(3) Stay organized and try to approach class methodically. Don't miss classes, quizzes assignments, etc. I try to keep things organized to help you do this.

(4) Do the suggested HW!
For each type of problem, ask yourself how comfortable you are with them.

If you can do a few easily <u>without</u> book/notes, great!

If not, use the book/notes, solve it, then <u>reflect on it</u>, then put a * next to it so you can come back tomorrow to try again without the notes.
(Example: the polynomial Q on E1)

(5) Use the resources available to you.
* Discussion
* Office Hours
* Help Desk!!!
* Tutoring Center
* Me!
* Studying / Working with classmates

Section 2.1: How do we measure speed?

Average Speed: Run 12 mi in 1.5 hours
your average speed is $\frac{12 \text{ mi}}{1.5 \text{ hr}}$
$= 8 \text{ mi/hr}$.

Instantaneous Speed: How fast were you going at exactly 3 minutes and 47 seconds?

If you had an extremely precise GPS/stopwatch combo, how could you estimate your instantaneous speed at 3 min, 47 sec? (227 seconds)

Calculate the average speed over some small interval around 227 seconds.

Calculate average speed from 217 → 237 sec.
$= \frac{\text{dist traveled in those 20 seconds}}{\text{20 seconds}}$

Average speed from 226.99 → 227.01

$$= \frac{\text{dist. trav. in those 0.2 sec}}{0.2 \text{ sec}}$$

As we use smaller and smaller windows, the estimate gets closer to closer to your true instantaneous speed.

Example from book: Throwing a Grapefruit

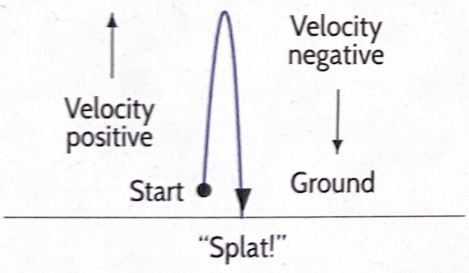


Table 2.1 *Height of the grapefruit above the ground*

t (sec)	0	1	2	3	4	5	6
y (feet)	6	90	142	162	150	106	30

"Velocity" is speed but it can be positive or negative.

Going up = positive velocity
Going down = negative velocity

Average ~~speed~~ velocity over the 1st second (0 sec → 1 sec)

$$\frac{\text{change in height}}{\text{change in time}} = \frac{90-6}{1} \quad \frac{84 \text{ ft}}{1 \text{ sec}} = 84 \text{ ft/sec}$$

Avg. vel. over 2nd second:

$$\frac{142-90}{2-1} = \frac{52 \text{ ft}}{1 \text{ sec}} = 52 \text{ ft/sec}$$

Avg. vel from 4 sec → 5 sec:

$$\frac{106-150}{5-4} = \frac{-44 \text{ ft}}{1 \text{ sec}} = -44 \text{ ft/sec}$$

* Suppose $s(t)$ is a function that tells you the position of an object at time t.

Average velocity from $t=a$ to $t=b$:

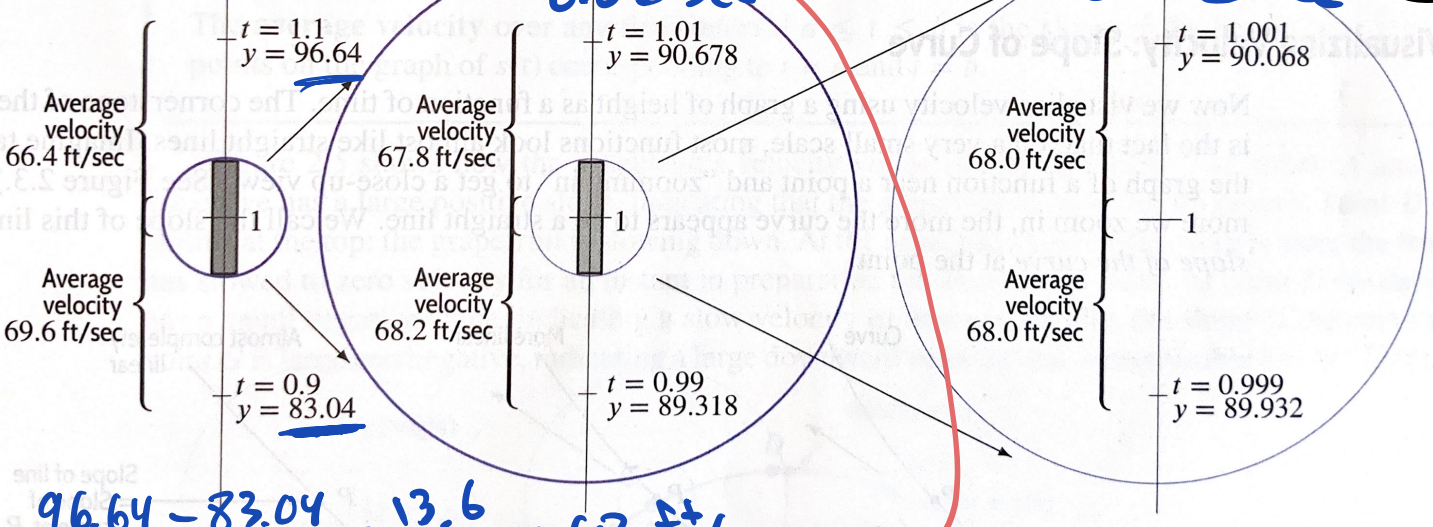
$$\text{Avg. Vel} = \frac{\text{change in pos.}}{\text{change in time}} = \frac{s(b)-s(a)}{b-a}$$

Let's estimate the instantaneous velocity of the grapefruit at $t=1$ using $s(t) = 6 + 100t - 16t^2$

0.9 sec → 1.1 sec

0.99 → 1.01
0.02 sec

0.999 → 1.001
0.002 sec



$\frac{96.64 - 83.04}{1.1 - 0.9} = \frac{13.6}{0.2} = 68$ ft/sec

$\frac{90.678 - 89.318}{0.02} = 68$ ft/sec

We are getting better and better estimates for the instantaneous velocity as the interval size shrinks more and more.

To calculate the inst. vel. at $t = a$:

* gap $= h = 1$ window $t = a \to t = a + 1$

avg. vel. $\frac{s(a+1) - s(a)}{(a+1) - a} = \frac{s(a+1) - s(a)}{1}$

* $h = 0.1$: $t = a \to t = a + 0.1$

$\frac{s(a+0.1) - s(a)}{(a+0.1) - a} = \frac{s(a+0.1) - s(a)}{0.1}$

* $h = 0.0001$: $t = a \to t = a + 0.0001$

$$\frac{s(a+0.0001)-s(a)}{(a+0.0001)-a} = \frac{s(a+0.0001)-s(a)}{0.0001}$$

We can't plug in $h=0$: $\frac{s(a)-s(a)}{a-a} = \frac{0}{0}$

Instantaneous velocity at $t=a$

$$* \quad \lim_{h\to 0} \frac{s(a+h)-s(a)}{h} \quad *$$

This is called a "derivative" one of the two main tools in all of calculus.

Example: Suppose the position of a car in feet after t seconds is given by $s(t)=t^2$. What is the instant. vel. of the car at $t=3$ seconds.

$$\text{I.V.} = \lim_{h\to 0} \frac{s(3+h)-s(3)}{h}$$

$s(3+h)$ — position at $3+h$ seconds

$$= \lim_{h \to 0} \frac{(3+h)^2 - 3^2}{h}$$

$$= \lim_{h \to 0} \frac{9 + 6h + h^2 - 9}{h}$$

$$= \lim_{h \to 0} \frac{h(6+h)}{h} = \lim_{h \to 0} (6+h) = 6 \text{ ft/sec.}$$