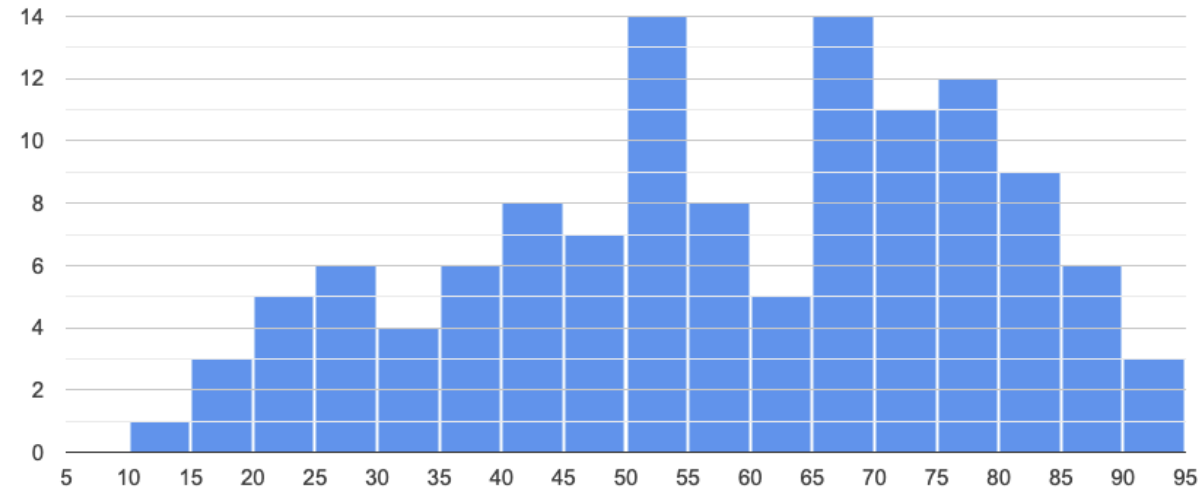
Mon, Oct. 3 - Fall '22
Lecture #15

Announcements / Reminders

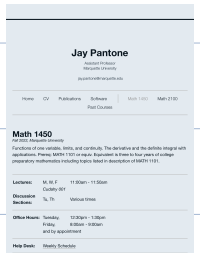
* Wiley Plus #5 due Wed (1.8 and 1.9)
* Quiz #4 on Thurs (1.8, 1.9, 2.1)
* Exam grades will be released later today. (out of 60)
  - Average ≈ 60%, pretty close to last time



- The precalc "review" is hard for a lot of people, and you might find Chapter 2 easier.
- If you score higher on Exam 2, I will increase your E1 grade up to the average of your E1 and E2 grades.
  Ex: E1: 30/60, E2: 40/60 E1: 30 → 35
- On Wednesday we will talk about tips for success

(Don't forget: Homework is worth 2.5 midterms)

Grade Calculator:



Jay Pantone

Home CV Publications Software Math 1450 Math 2130 Past Courses

Math 1450

Fall 2022, Marquette University

Functions of one variable, limits, and continuity. The derivative and the definite integral with applications. Prereq: MATH 1101 or equiv. Equivalent to three to four years of college preparatory mathematics including topics listed in description of MATH 1101.

Lectures: M, W, F 11:00am – 11:50am Cudahy 491

Discussion Sections: Tu, Th Various times

Office Hours: Tuesday 12:30pm – 1:30pm Friday 8:00am – 9:00am and by appointment

Help Desk: Weekly Schedule

2

Section 1.9: Further Limit Calculations

(3) Last type: $\lim_{x \to c} \frac{f(x)}{g(x)}$ when $g(c) = 0$.

Ex: $\lim_{x \to 3} \frac{x+1}{x-3}$ What happens near 3?

Numer: Near 4 } function blows up
Denom: Near 0 } to $+\infty$ or $-\infty$

But does it blow up to $+\infty$ or $-\infty$ on each side?

$\lim_{x \to 3} \frac{x+1}{x-3}$

Case 1: $x \to 3^+$

Numer $\to 4^+$ (getting #s slightly bigger than 4)

Denom $\to 0^+$

$x = 3.0001 \Rightarrow$ Numer $= 4.0001$ / Denom $= 0.0001$

Is $\lim_{x\to 3^+} \frac{x+1}{x-3}$ $+\infty$ or $-\infty$.

because we're getting $\frac{+}{+}$

Case 2: $x \to 3^-$ $(x = 2.999)$

Numer $\to 4^-$

Denom $\to 0^-$ $(2.999 - 3 = -0.001)$

$\lim_{x\to 3^-} \frac{x+1}{x-3}$ $+\infty$ or $-\infty$

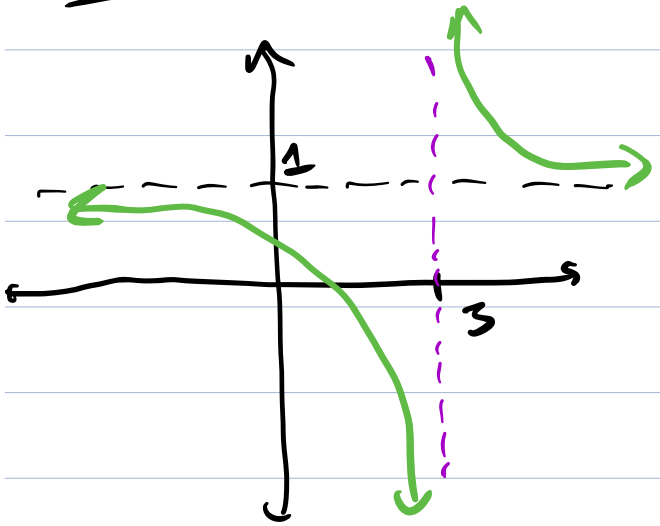
$\frac{\text{numer}}{\text{denom}} = \frac{\text{positive}}{\text{negative}} = \text{negative}$

$\lim_{x\to\infty} \frac{1\cdot x+1}{1\cdot x-3} = 1$

$\lim_{x\to 3^-} \frac{x+1}{x-3} = -\infty$

$\lim_{x\to 3^+} \frac{x+1}{x-3} = +\infty$

$\lim_{x\to 3} \frac{x+1}{x-3} = \text{DNE}$



General Rule: $\lim_{x \to c} \frac{f(x)}{g(x)}$

If (1) $f(x)$ is continuous at $x = c$ (all polynomials qualify)

(2) $f(c) \neq 0$

(3) $g(c) = 0$

then the one-sided limits of $\frac{f(x)}{g(x)}$ as $x \to c$ must be $+\infty$ or $-\infty$.

→ To figure out which, think about whether f and g are positive or negative on each side.

The last case is when the numer. and denom. are both 0 at $x = c$.

Technique: Do arithmetic, try to factor and cancel. The limit might exist or might not exist.

Ex: $\lim_{x \to 3} \frac{x^2 - x - 6}{x - 3}$ plug in 3 gives $\frac{0}{0}$

$$= \lim_{x \to 3} \frac{(x-3)(x+2)}{(x-3)} = 5$$

Ex: $\lim_{h\to 0} \dfrac{(3+h)^2-3^2}{h}$ $\quad \dfrac{0}{0}$

$= \lim_{h\to 0} \dfrac{9+6h+h^2-9}{h}$

$= \lim_{h\to 0} \dfrac{h\cdot(6+h)}{h} = 6+0 = 6$

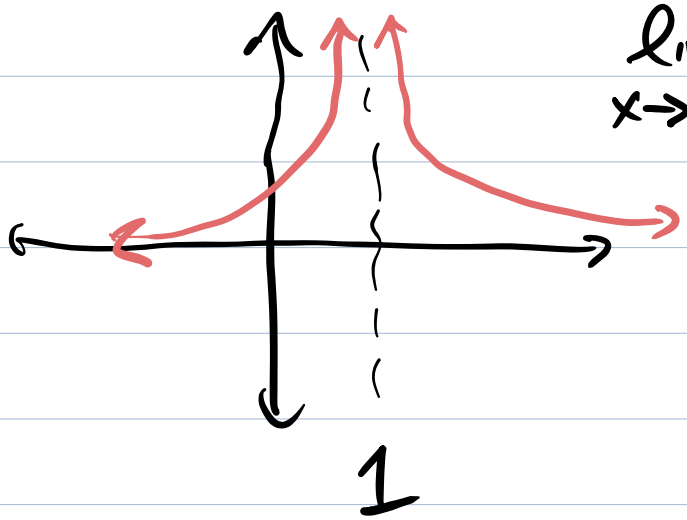
Ex: $\lim_{x\to 1} \dfrac{x-1}{x^2-2x+1}$ $\quad \dfrac{0}{0}$

$= \lim_{x\to 1} \dfrac{x-1}{(x-1)(x-1)}$

$= \lim_{x\to 1} \dfrac{1}{x-1}$ $\quad \dfrac{1}{0}$

Now we use technique (3) to figure out if it's $\pm\infty$

$\dfrac{1}{(x-1)^2}$



$\lim_{x\to 1} \dfrac{1}{(x-1)^2} = +\infty$

Squeeze Theorem

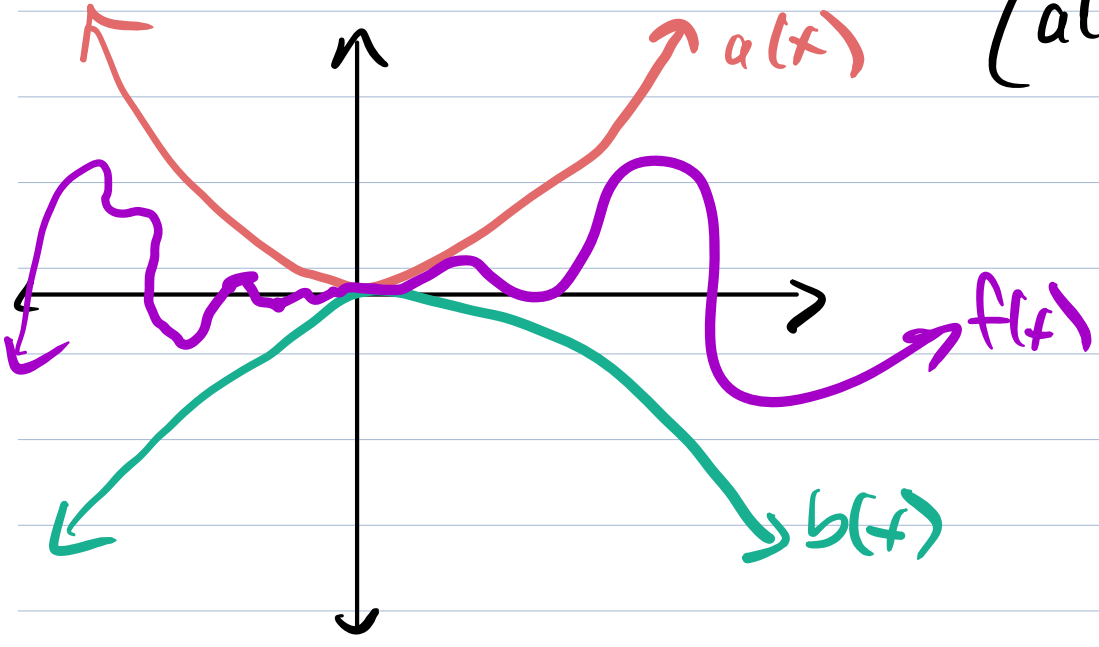
Suppose you have two functions $a(x)$ and $b(x)$ and that $a(x) \geq b(x)$ for all x-values.

Suppose that there is some x-value c where

$$\lim_{x \to c} a(x) = \lim_{x \to c} b(x) = L.$$

Then: if $f(x)$ is some function that is always <u>between</u> $a(x)$ and $b(x)$, then

$$\lim_{x \to c} f(x) = L.$$



($a(x)$ and $b(x)$ meet at $(0,0)$)

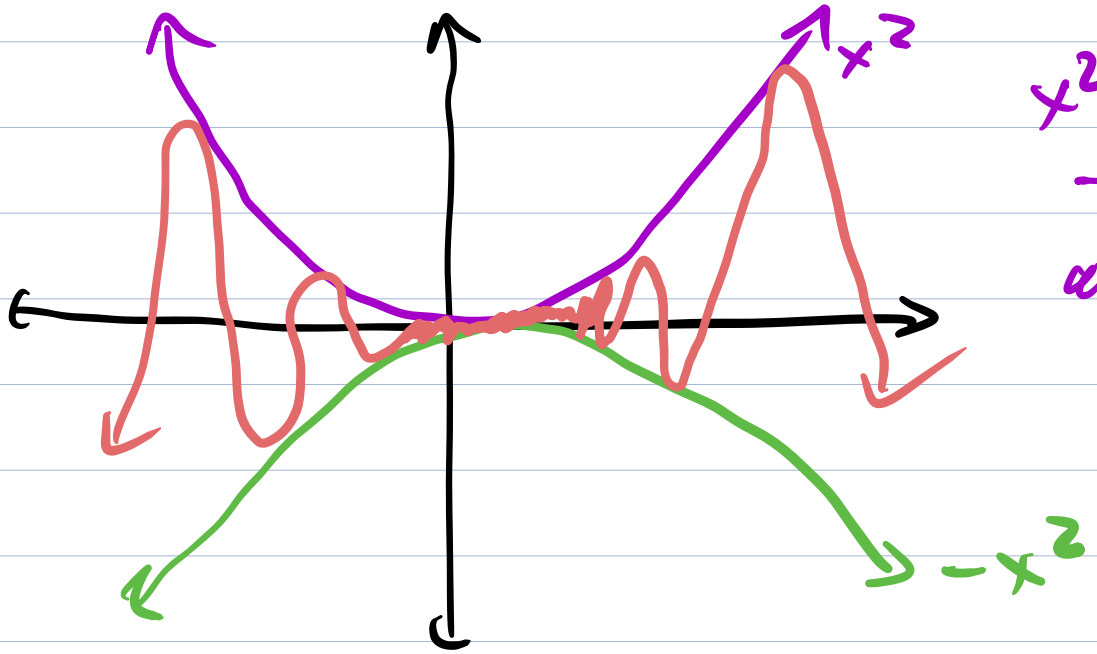
$\lim_{x \to 0} f(x) = 0$

Ex: $\lim_{x \to 0} x^2 \cos\left(\frac{1}{x}\right)$

$$-1 \leq \cos(\text{anything}) \leq 1$$

$$-1 \leq \cos\left(\frac{1}{x}\right) \leq 1$$

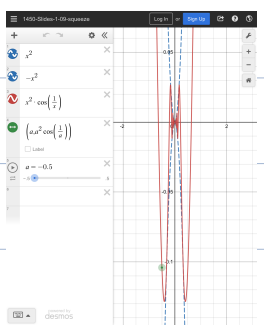
$$-x^2 \leq x^2 \cos\left(\frac{1}{x}\right) \leq x^2$$



x^2 and $-x^2$ meet at $x=0$

So by the squeeze theorem

$$\lim_{x \to 0} x^2 \cos\left(\frac{1}{x}\right) = 0$$



Section 2.1: How Do We Measure Speed?

Average Speed is an easy concept.

If you ran 12 miles in 1.5 hours, then your average speed

$$\frac{12 \text{ mi.}}{1.5 \text{ hr}} = 8 \frac{\text{mi}}{\text{hr}}$$

Instantaneous Speed is weirder

How fast were you going at exactly 3 minutes and 47 seconds.

(How do speedometers do it?)