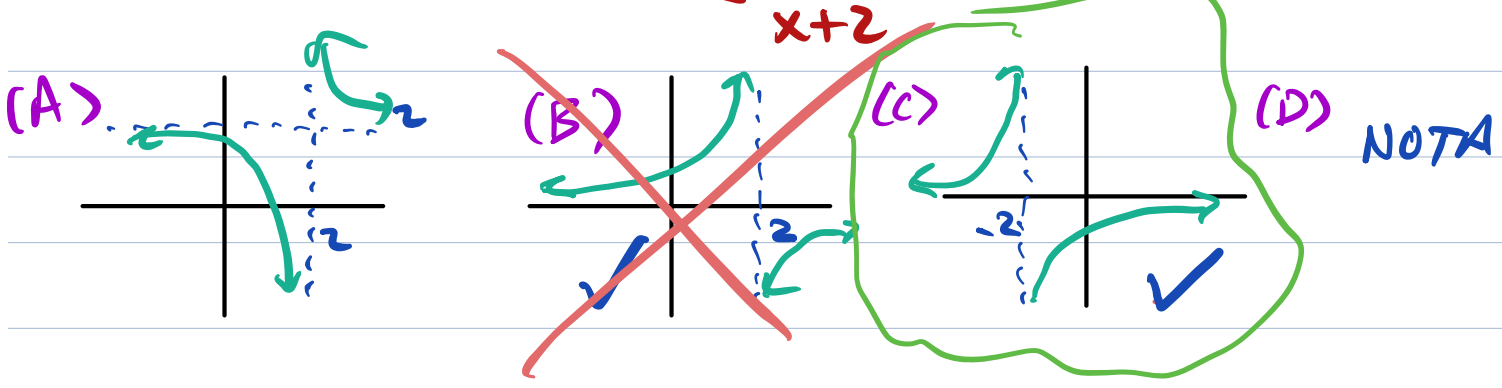
Friday, Sept. 21 - Fall '22
Lecture #11

Announcements / Reminders

* Today: covering 1.7
* Monday: 1.7 + some of 1.8
* Tuesday: Discussion + Office Hours 12:30 - 1:30
* Wednesday: Exam 1 in class
  covers all material up to and including Mon, Sept. 26
  Wiley Plus HW 4 due at 11:59pm
  (1.6, 1.7, some of 1.8)
* Thursday: Discussion
  Quiz 4 - 1.7, some of 1.8
* Friday: Lecture

(2) Sketch as much as you can of the rational function $-\frac{4}{x+2}$.



Section 1.7 - Introduction to Limits and Continuity

All of Calculus is built on the concept of a limit.

"How does a function behave as you get closer and closer and closer to a point?"

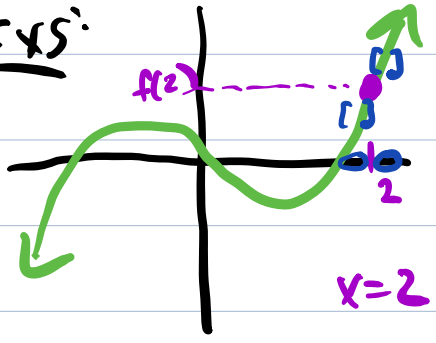
Continuity

A function is continuous at a point if:

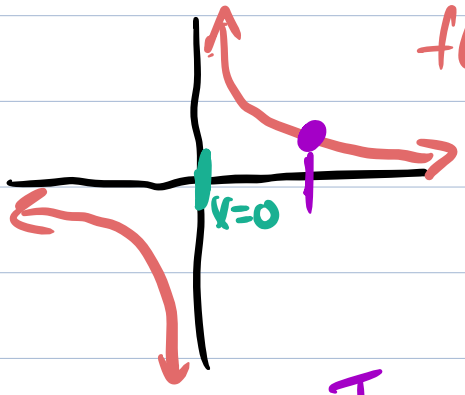
(1) it exists at the point
(2) there are no jumps or breaks at the point

A function is continuous on an interval if it's continuous at every point in the interval.

Exs:



This function is continuous at every point.



Not continuous at $x=0$.
(Doesn't exist at $x=0$)

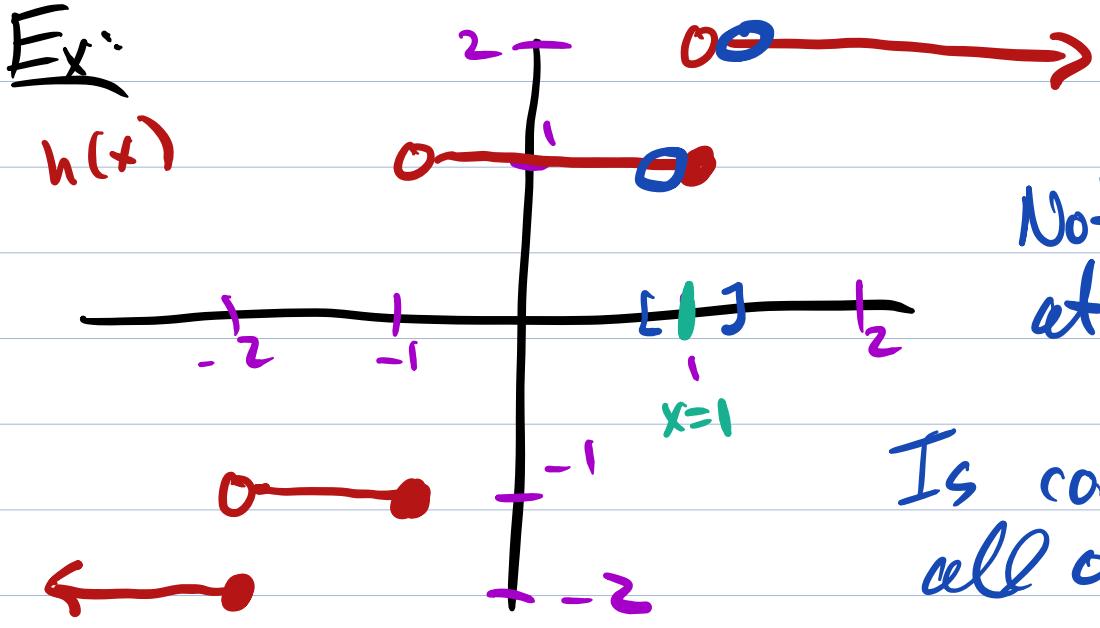
Is continuous at $x=2$ ✓

Is continuous on the interval $(0, \infty)$

$(-\infty, 0)$

$[6, 7]$

Ex:



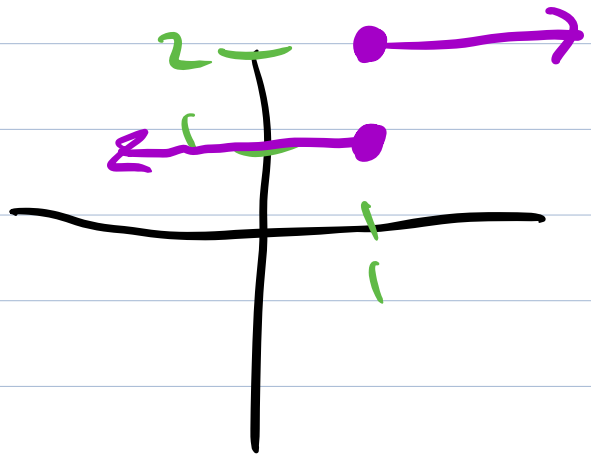
Not continuous at $x=-2, -1, 1$.

Is continuous at all other points.

$h(-2) = -2$ not -1

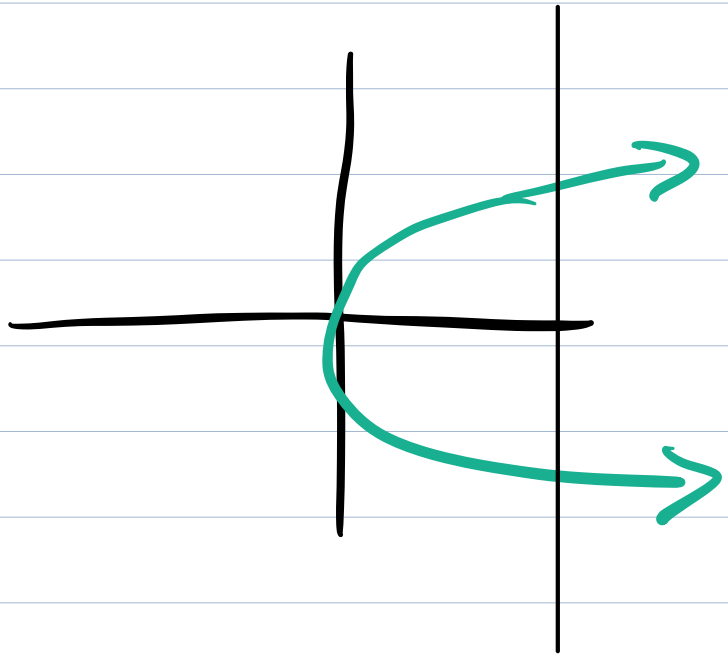
$(-\infty, -2)$ $(-2, -1)$ $(-1, 1)$

$(1, \infty)$



$f(1) = 1$

$f(1) = 2$



Another way to say "f(x) is continuous at the point $x = c$" is:

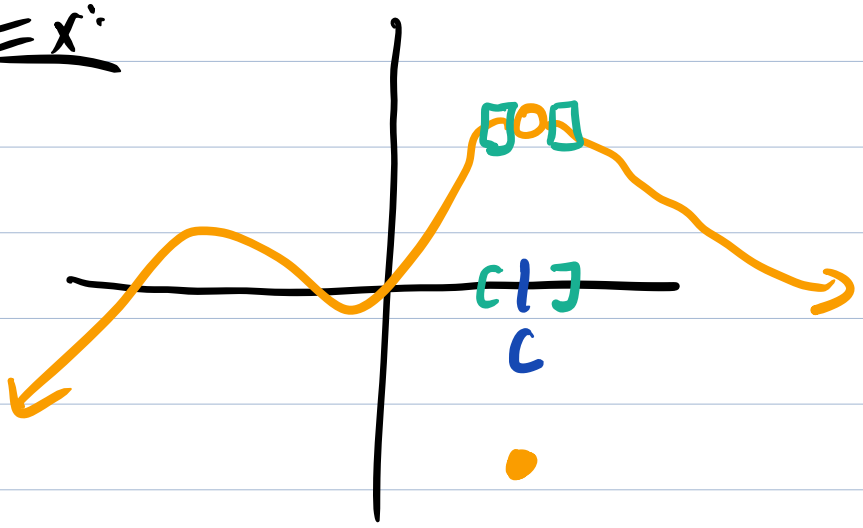
"The y-values of f(x) when x is very close to c are very close to f(c)"

rephrased:

"When x is very close to c, the y-values of f(x) are very close to f(c)."

Rephrased:
"As x approaches c, f(x) approaches f(c)"

Ex:



Not continuous at $x=c$.

Which functions are continuous?

- Exponentials
- Polynomials
- Sine + Cosine
- Logarithms
- Rational Functions
- Tangent

are continuous at all points where they are <u>defined</u>

<u>Limits</u> "$\lim_{x \to c} f(x)$"

= "the limit of f(x) as x approaches c"
= what does it look like f(c) <u>should equal</u>

(1) looking at the x-values

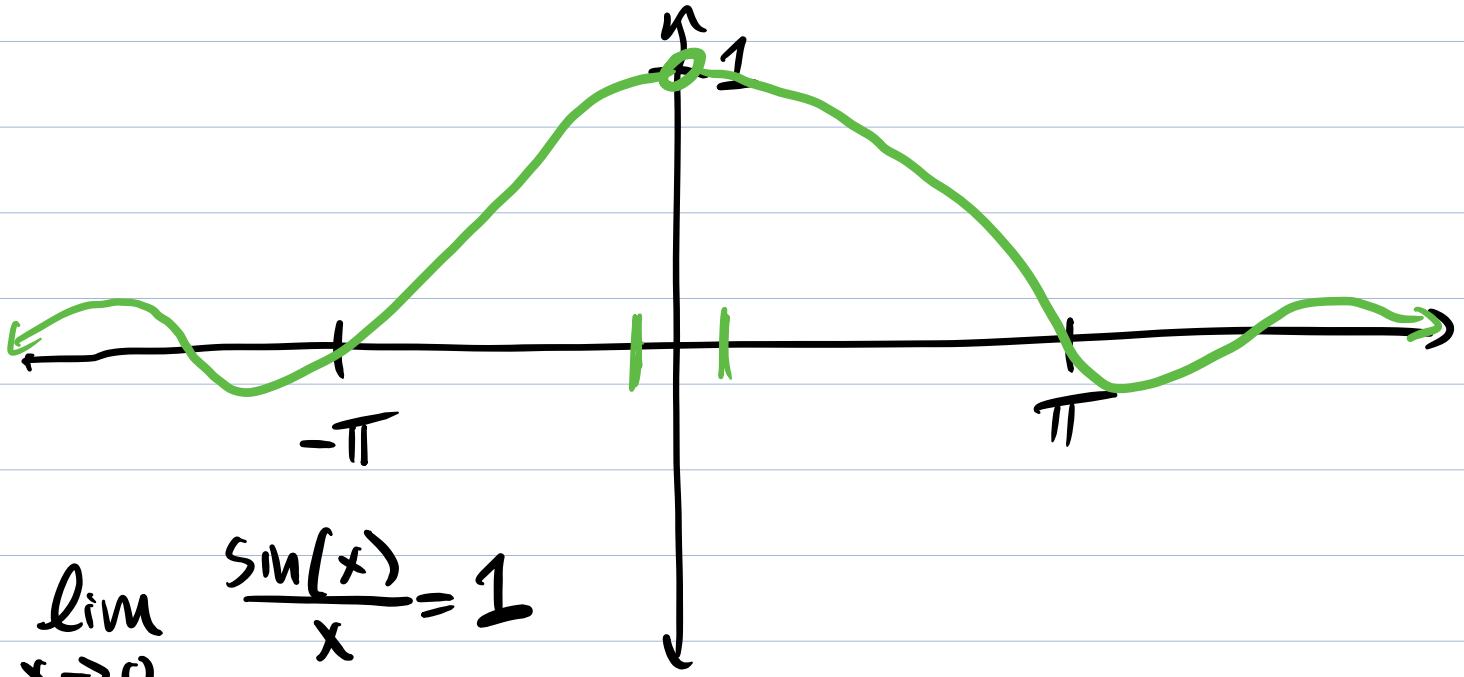
close to c

(2) ignoring f(c)

Ex: Let $f(x) = \frac{\sin(x)}{x}$

f(0) doesn't exist

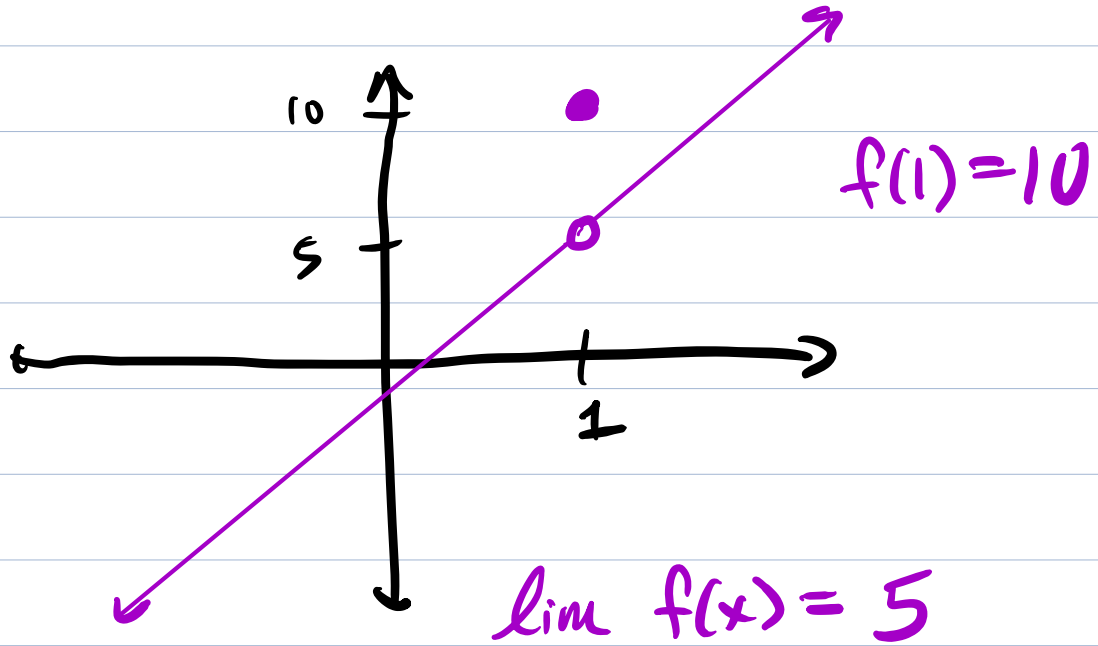
But if it did, what should it be?



$$\lim_{x \to 0} \frac{\sin(x)}{x} = 1$$

Ex:

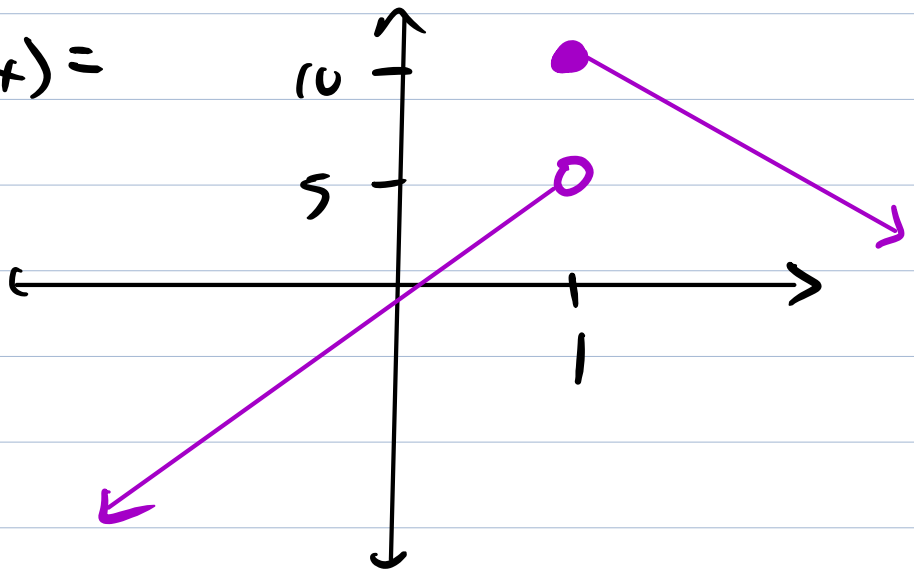
$f(x) =$



$$\lim_{x \to 1} f(x) = 5$$

The limit doesn't care what actually happens at $x=1$, just nearby.

Ex: Let $f(x) =$

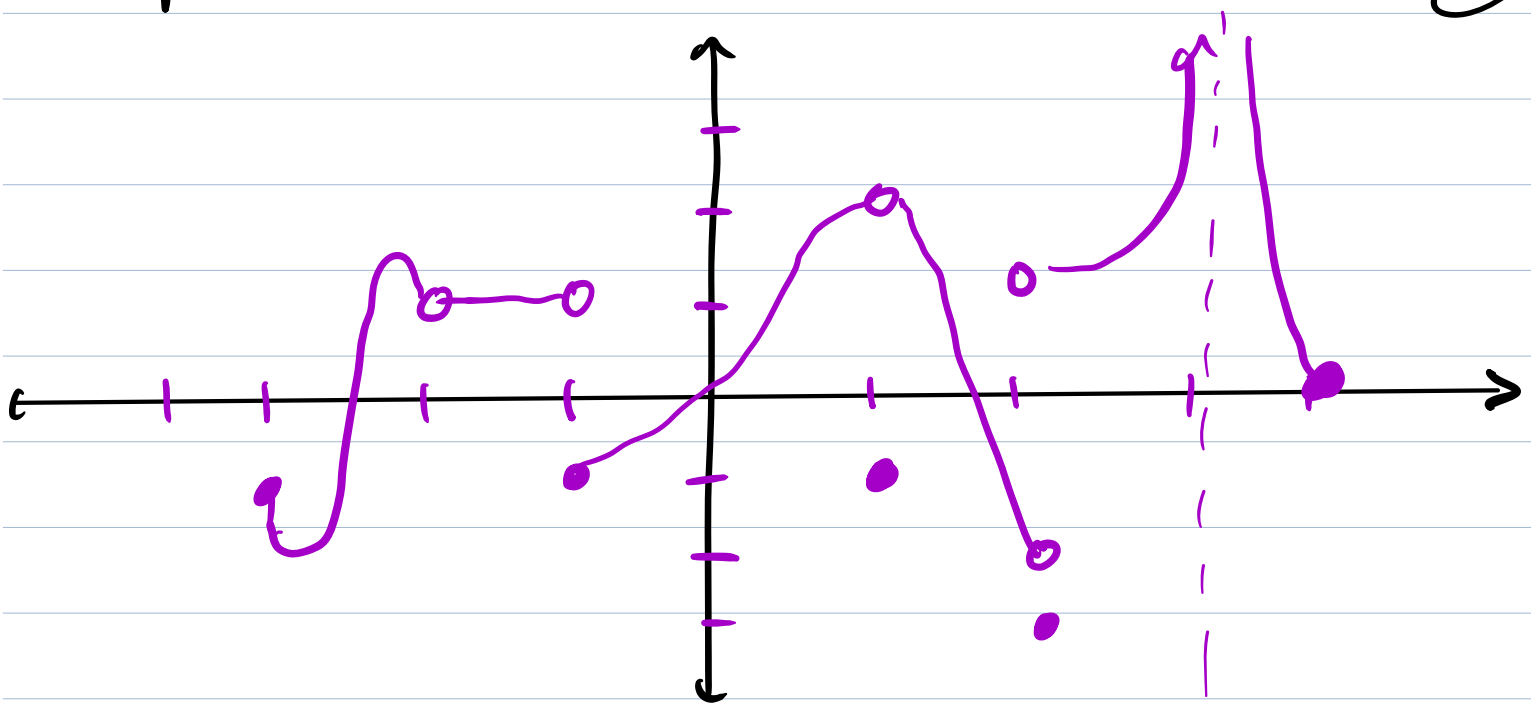


What is $\lim_{x \to 1} f(x)$?

- One side makes it look like it should be 5
- The other side suggests 10
- Ambiguous, so the limit Does Not Exist (DNE)

$\lim_{x \to 1} f(x) = \text{DNE}$

Group Work:



Find the limits at

$x = -3, -2, -1, 1, 2, 3$.

DNE ↓ 1 ↓ DNE ↓ 2 ↓ DNE ↪ DNE ∞