Wednesday, Sept. 21 - Fall '22
Lecture #10

Announcements / Reminders

* WP HW 3 due tonight 1.4, 1.5, some 1.6
* Q3 tomorrow 1.5, 1.6 (sugg HW from Fri, Mon, today)
* E1 on Wednesday 9/28
  ↳ material up to and including Monday 9/26
* ODS Proctoring
* Help Desk! Office Hours!
  ↳ Wed 12-1

→ Power Functions $k \cdot x^p$ $5x^{-2}$

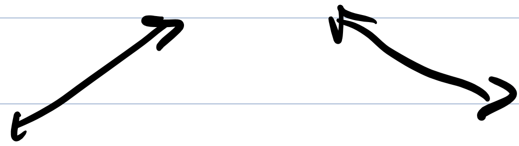
→ Polynomials

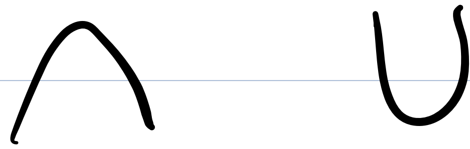
$$p(x) = \underbrace{-3x^{4} + 5x^2 - 2x + 1}_{\text{coefficients}}$$

degree

Fact: A degree n polynomial can turn around at most n-1 times.

Examples

degree 1: 

degree 2: 

degree 3: 

0 2

0 times
2 times

degree 4: 

1 time
3 times

degree 5: 

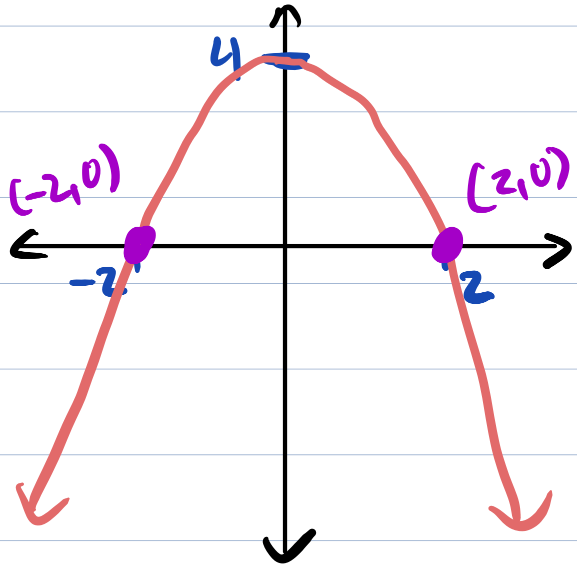
0 2 4

0 times
2 times
4 times

x-intercepts: If a polynomial $p(x)$ touches the x-axis at a point $x=c$, then

$(x-c)$ must be a factor of $p(x)$.

Ex:



Can we find a degree 2 polynomial that looks like this?

Has x-intercepts at $x=-2$ and $x=2$

The polynomial has a factor of $\underbrace{(x-(-2))}_{(x+2)}$ and $(x-2)$.

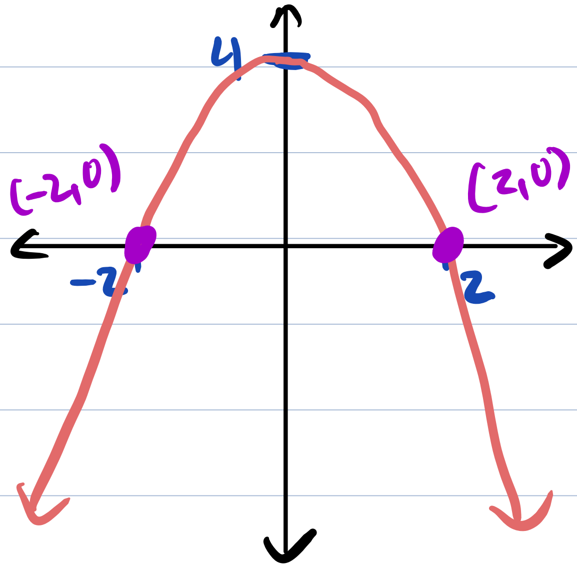
$p(x) = ???\,(x-2)\cdot(x+2)\cdot\, ???$

Is it possible that $p(x)$ has more factors like $(x-2)(x+2)(x+1)\cdots$

No because this would give degree three or higher.

What we're still missing is the possibility of a constant in front.

$p(x) = k\cdot(x-2)(x+2)$



Since the graph passes through (0,4) we know $p(0)=4$.

$4 = k\cdot(0-2)\cdot(0+2)$

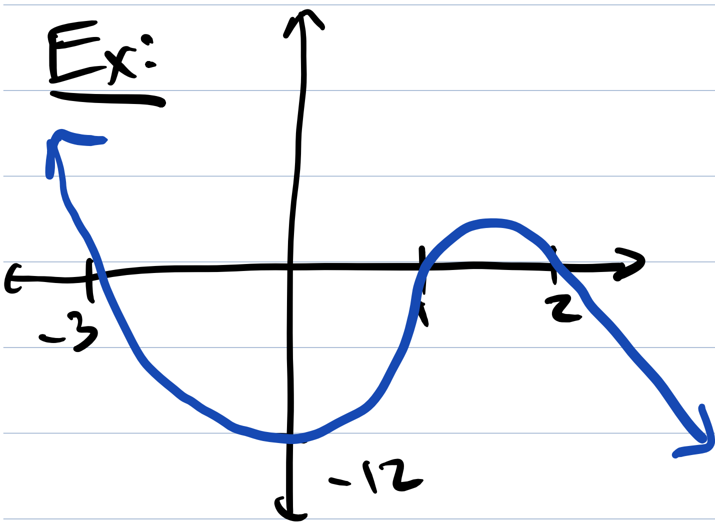
$4 = k\cdot(-4)$

$\Rightarrow k=-1.$

$$\boxed{p(x) = -(x-2)(x+2)}$$

$p(x) = -(x^2+2x-2x-4) = -x^2+4$

Ex:



Can we find a cubic (deg. 3) poly that looks like this?

x-intercepts: $-3, 1, 2$

factors: $(x+3), (x-1), (x-2)$

$p(x) = k\cdot(x+3)(x-1)(x-2)$

$p(0) = -12$

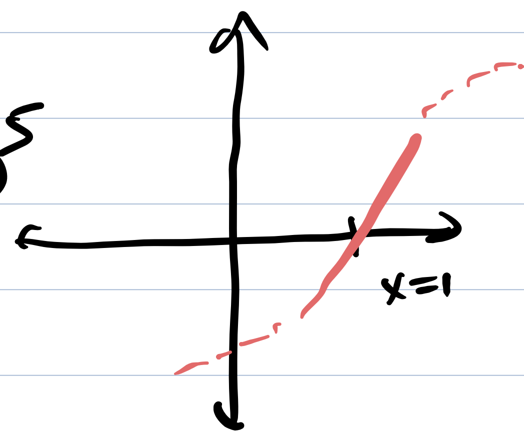
$-12 = k\cdot 3\cdot(-1)\cdot(-2)$

$\Rightarrow -12 = k\cdot 6 \quad \Rightarrow \boxed{k=-2}$

$p(x) = -2(x+3)(x-1)(x-2)$

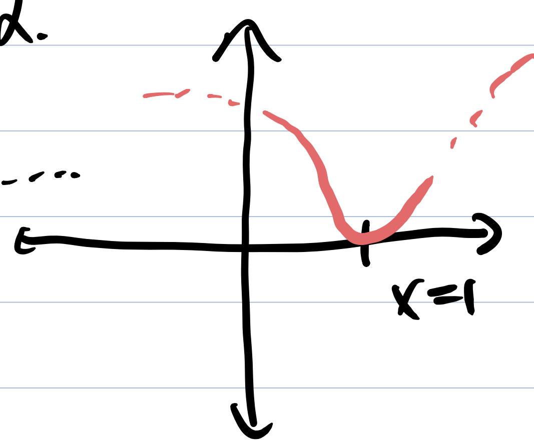
Fact:
If a polynomial touches the x-axis at $x=c$ and crosses through it then the polynomial has a factor of $(x-c)^k$ where k is an odd #.

$(x-1)$ or $(x-1)^3$ or $(x-1)^5$

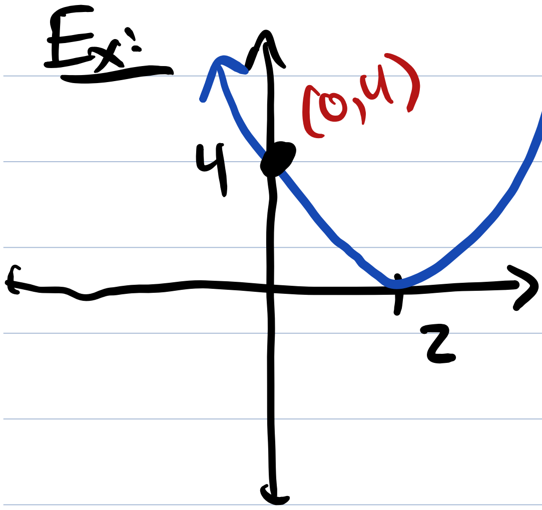


Fact:
If a poly. touches the x-axis at $x=c$ and bounces off, then the polynomial has a factor of $(x-c)^k$ where k is an even # ≥ 2.

$(x-1)^2$ or $(x-1)^4$ or $(x-1)^6$ ....



Ex:



Find the degree 2 poly. that looks like this.

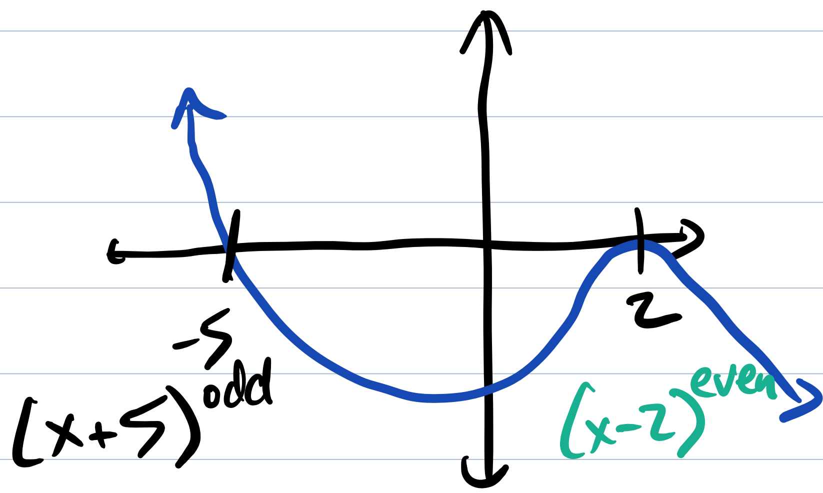
Because it bounces off we have a factor of $(x-2)^2$.

$p(x) = k \cdot (x-2)^2$

$4 = k \cdot (-2)^2 \Rightarrow 4 = k \cdot 4$

$\Rightarrow k = 1$

$p(x) = (x-2)^2$



degree 3

degree 5 ?

Rational Function

Def: A rational function is just a fraction of polynomials

$$r(x) = \frac{p(x)}{q(x)}$$ where p and q are polynomials

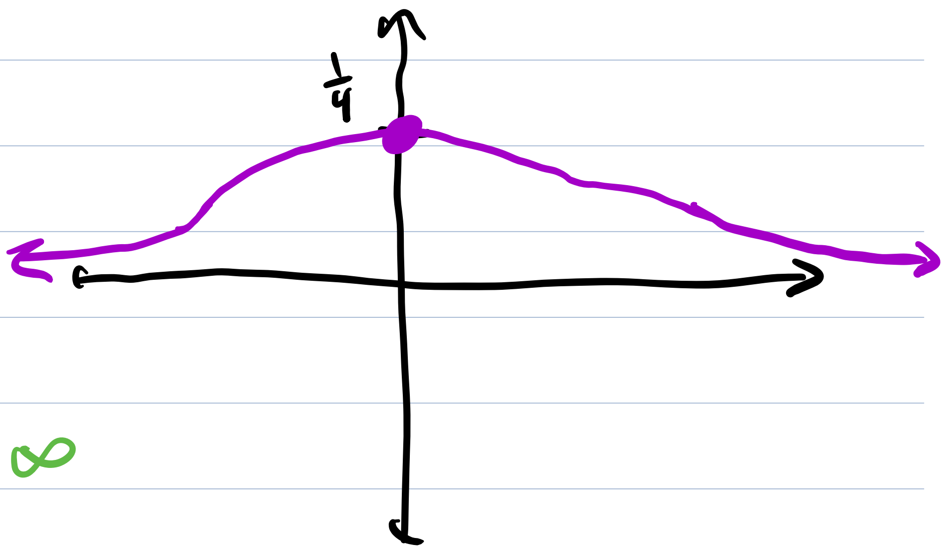
They are hard to graph in some cases, but not always.

Ex: $$f(x) = \frac{1}{x^2+4}$$ "1" is a degree 0 polynomial

What can we deduce about $f(x)$?

$$f(0) = \frac{1}{0^2+4} = \frac{1}{4}$$

What happens to $f(x)$ as $x \to \infty$?



$$\frac{1}{(BPN)^2+4} = RSPN$$

$f(x) \to 0$ as $x \to \infty$

As $x \to -\infty$

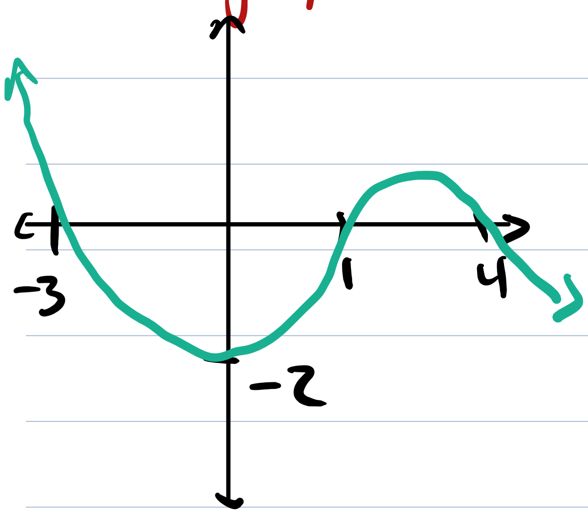
$$\frac{1}{(BNN)^2+4} = RSPN$$

$f(x) \to 0$ as $x \to -\infty$

The book has many many more examples.

Group work:

(1) Find a cubic poly. that matches the graph



(A) $\frac{1}{6}(x+3)(x-1)(x-4)$

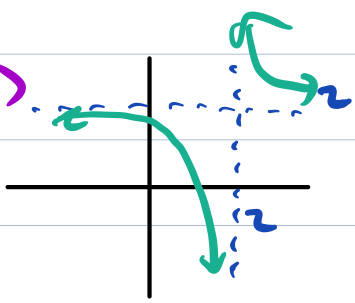
(B) $-\frac{1}{6}(x-3)(x+1)(x+4)$

(C) $-\frac{1}{6}(x+3)(x-1)(x-4)$ ✓

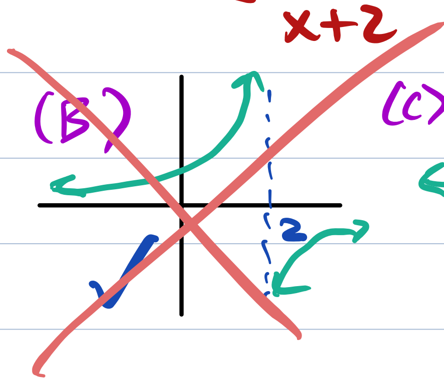
(D) NOTA

(2) Sketch as much as you can of the rational function $-\frac{4}{x+2}$.

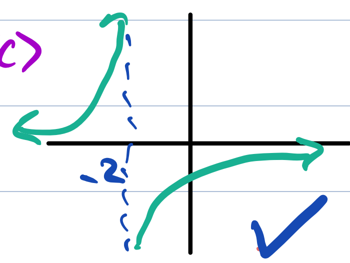
(A)



(B)



(C)



(D) NOTA