Monday, Sept. 19 - Fall '22
Lecture #9

Announcements / Reminders

* WP HW 3 due Wed — 1.4, 1.5, some 1.6
* Q3 on Thursday — 1.5, 1.6
* E1 on 9/28
  ↳ material up to and including Monday 9/26

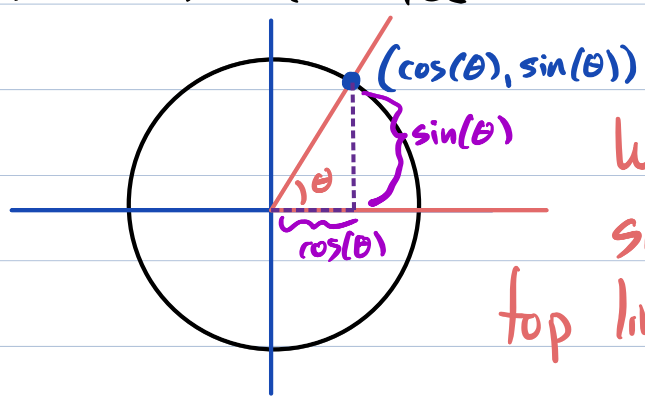
only 34/125 have started

Section 1.5 — Trig Functions

Tangent:
The function $\tan(\theta)$ is defined as

$$\tan(\theta) = \frac{\sin(\theta)}{\cos(\theta)}$$

It also has a nice visual interpretation:

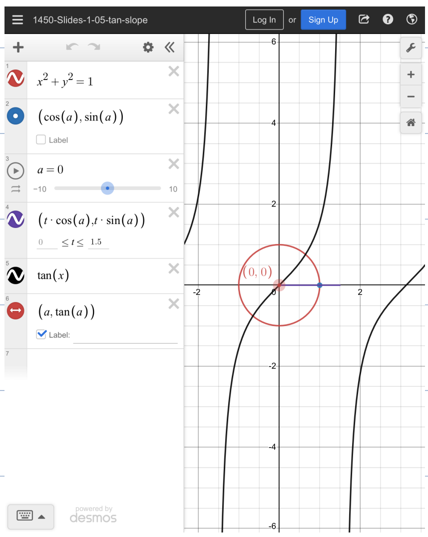


What is the slope of the top line?

$\frac{\text{rise}}{\text{run}} = \frac{\sin(\theta)}{\cos(\theta)} = \tan(\theta)$ (2)

So, $\tan(\theta)$ is the slope of the line that the angle makes.

$\tan(0) = 0 \quad \tan\left(\frac{\pi}{4}\right) = 1 \quad \tan\left(\frac{\pi}{2}\right) = \text{undefined}$



amplitude = not defined

period = π

Skipping: (1) csc, sec, cot
(2) inverse trig functions (for now)
arcsin or $\sin^{-1}$

Section 1.6 – Power, Polynomials, and Rational Functions

Def: A <u>power function</u> is a function of the form

$$f(x) = k \cdot x^p$$

constants

(compare to exponential functions:

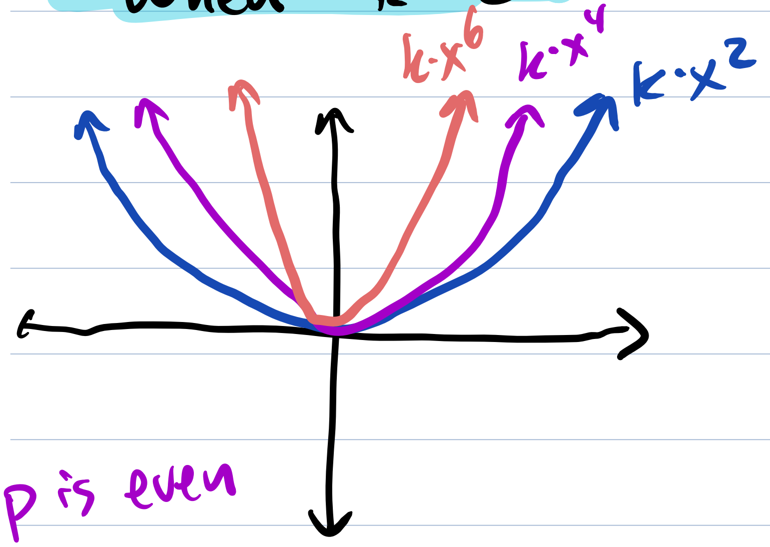
$$f(x) = k \cdot a^x$$)

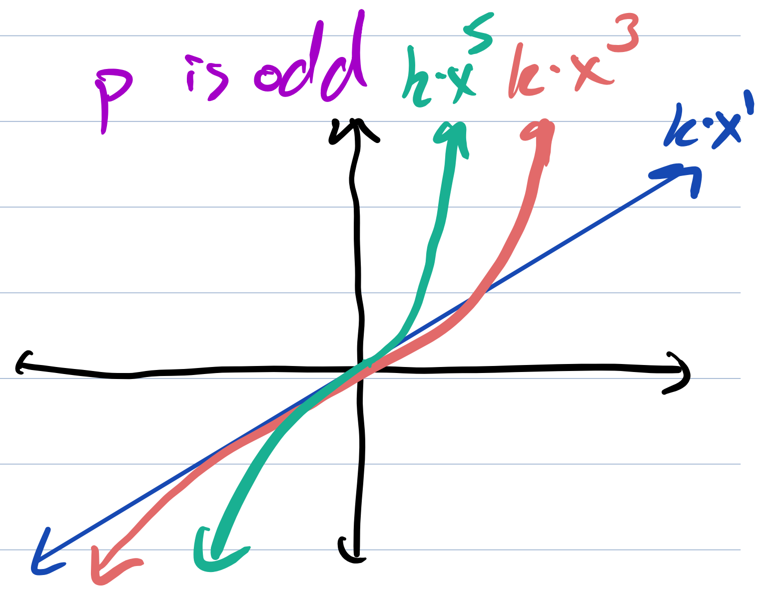
Examples of power functions:

$3x^2 \qquad \frac{1}{2}x^{-3} \qquad -5x^{10} \qquad \pi x^{1/2}$

How to graph $k \cdot x^p$ when p is a positive whole #.

When $k > 0$:





When $k<0$:
Flip all these pictures upside down.

How do we know these shapes?

<u>Example:</u> $f(x) = -3x^7$

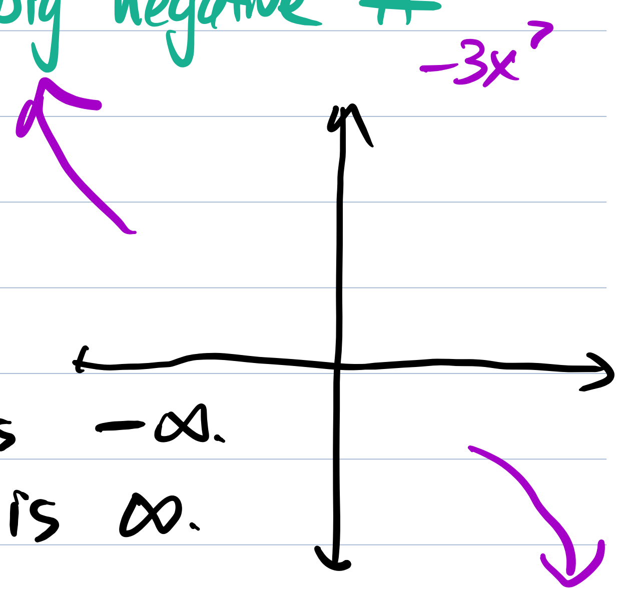
<u>Q:</u> What is the behavior of the graph as we go off to the left and right?

Rephrased: What is the "limit" of $f(x)$ as $x \to \infty$?
What is the "limit" of $f(x)$ as $x \to -\infty$?

$f(x) = -3x^7$

$f(\text{big positive number}) = -3\cdot(BPN)^7$
$= -3\cdot(EBPN)$
$=$ big negative #

$f(\text{big negative number})$
$= -3\cdot(BNN)^7$
$= -3\cdot(EBNN)$
$=$ big positive #



The limit of $f(x)$ as $x \to \infty$ is $-\infty$.
The limit of $f(x)$ as $x \to -\infty$ is ∞.

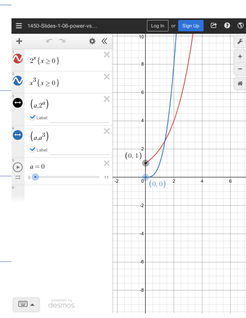
$(-2)^2 = (-2)\cdot(-2) = 4$

$(-2)^3 = (+2)\cdot(+2)\cdot(-2) = -8$

Power Functions vs. Exponential Functions

$k \cdot x^p$ $\qquad$ $k \cdot a^x$

grows way faster (eventually)



Polynomials

Def: A polynomial is a bunch of power functions (with non-negative whole # powers) added together.

powers

Ex: $p(x) = -3x^4 + 5x^2 - 2x^1 + 1\cdot x^0$

constants / coefficients

This polynomial has no x^3 term and that's fine. $(0\cdot x^3)$

General Form:

$$p(x) = a_n x^n + a_{n-1} \cdot x^{n-1} + a_{n-2} \cdot x^{n-2} + \dots + a_1 x + a_0$$

For our last example:

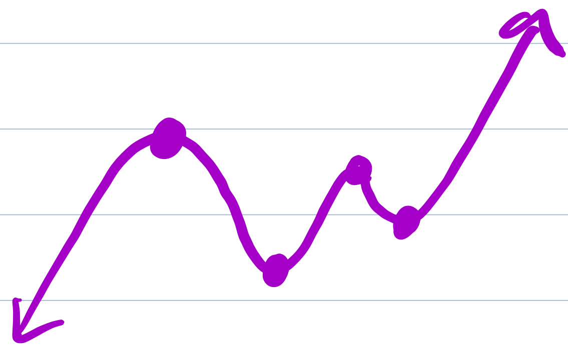
$n=4$, $a_4=-3$, $a_3=0$, $a_2=5$, $a_1=-2$, $a_0=1$

n is called the "degree" of the polynomial (the highest power of x with a nonzero coefficient)

General Shape:

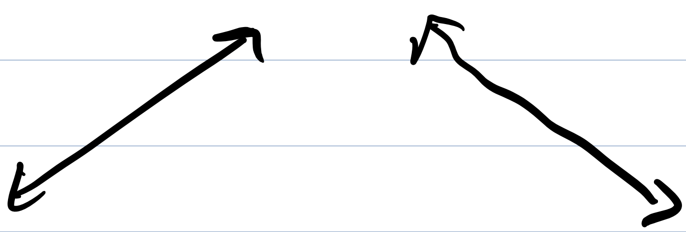
A degree n polynomial can "turn around" up to $n-1$ times.

$n=5$:



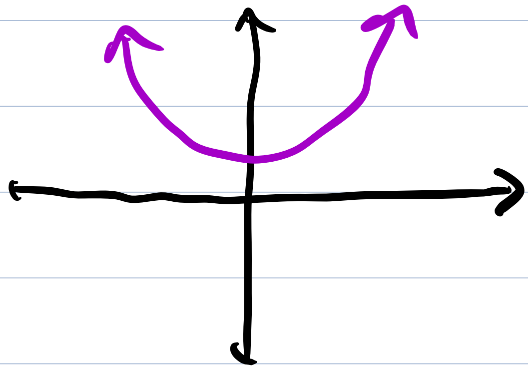
turns around 4 times

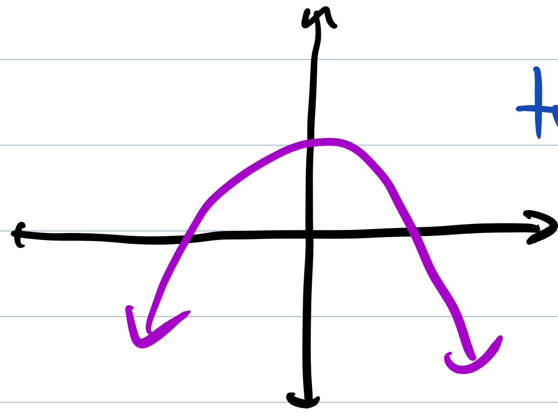
$n=1$: $a_1 x + a_0$ $(mx+b)$



turn around 0 times

n=2: $a_2x^2 + a_1x + a_0$

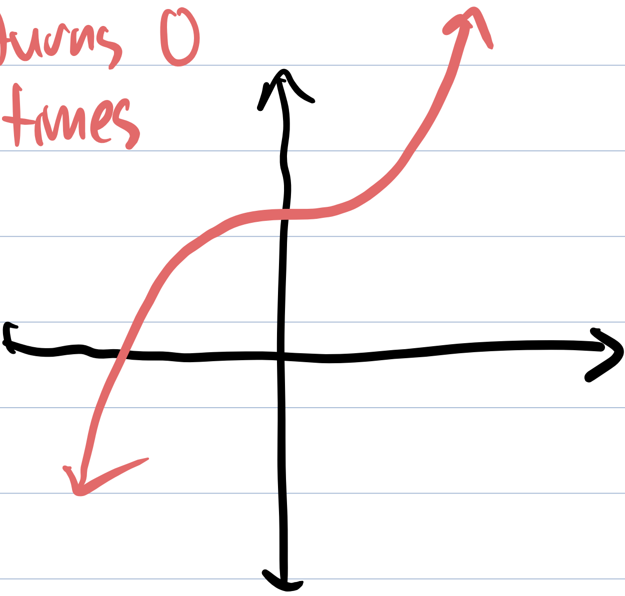




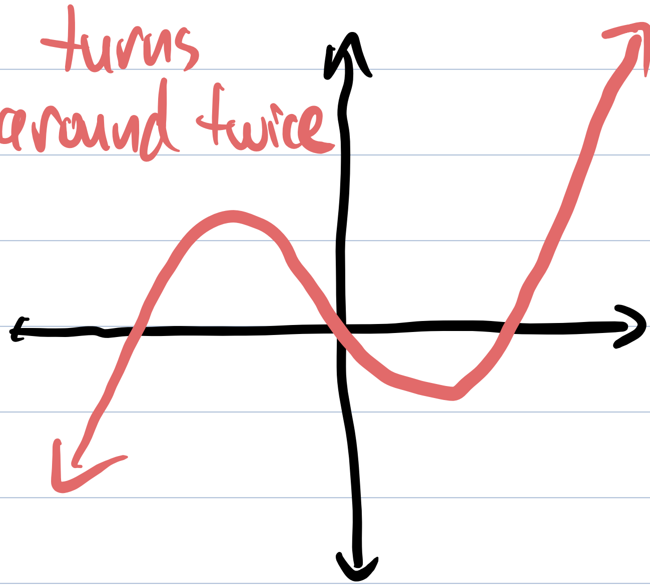
turn around once

n=3: $a_3x^3 + a_2x^2 + a_1x + a_0$

turns 0 times



turns around twice



n=4 turn around 1 or 3 times