

Friday, Sept. 16 - Fall '22
Lecture # 8

(1)

Announcements / Reminders

- * WP HW 3 due next Wed 1.4, 1.5, some 1.6
- * Q3 next Thursday 1.5, 1.6
- * E1 on 9/28

Section 1.5 - Trigonometric Functions

Two units of measure for angles
radians and degrees

A circle has 360° or 2π radians

So the conversion factor is

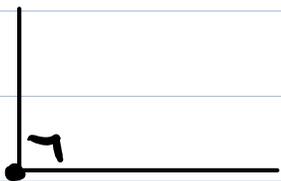
$$\text{degrees} = \frac{360}{2\pi} \cdot \text{radians}$$

$$= \frac{180}{\pi} \cdot \text{radians}$$

The other direction:

$$\text{radians} = \frac{\pi}{180} \cdot \text{degrees}$$

Exs:



"right angle"

(2)

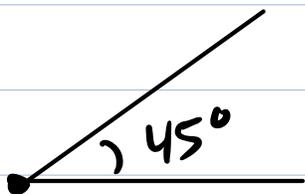
90°

$$\left(\frac{360}{4}\right)$$

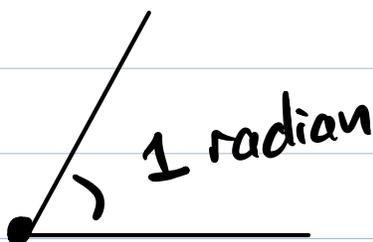
How many radians?

$$\frac{\pi}{180} \cdot 90 = \frac{\pi}{2} \text{ radians}$$

$$\approx \frac{3.1}{2} \approx 1.5$$



$$45^\circ = \frac{\pi}{4} \text{ radians}$$



convert to degrees:

$$\frac{180}{\pi} \cdot 1 = \left(\frac{180}{\pi}\right)^\circ$$

$$\approx 57.3^\circ$$

"acute" = less than 90°

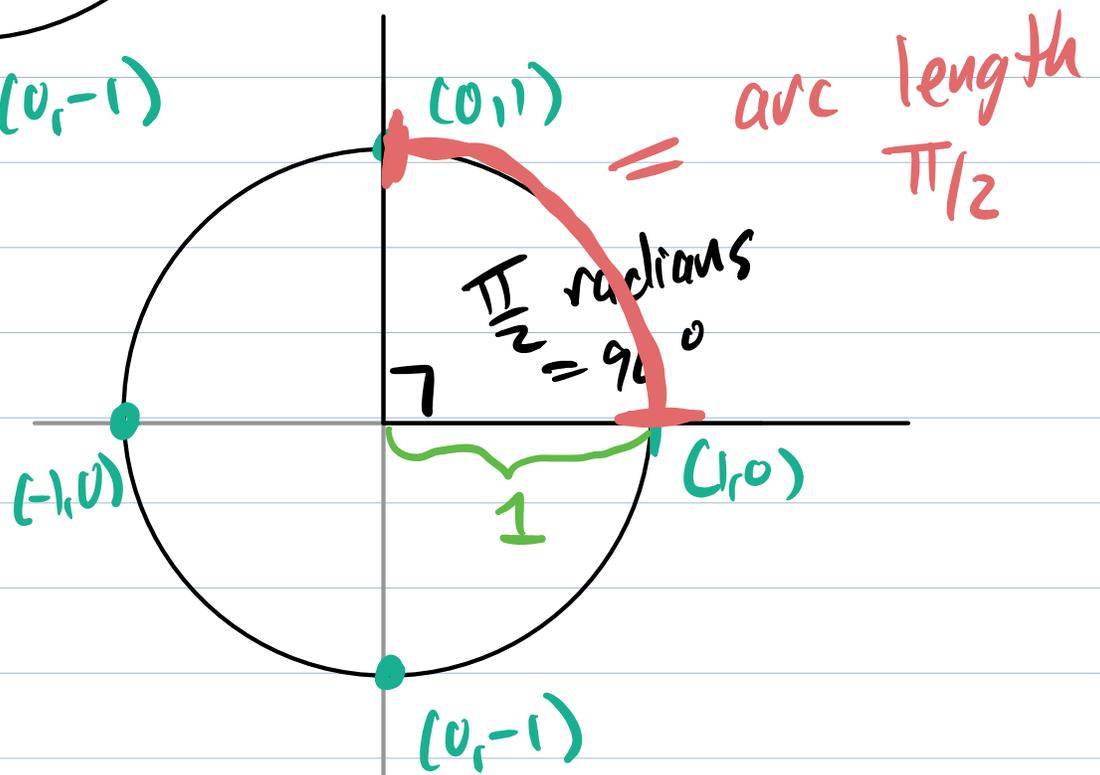
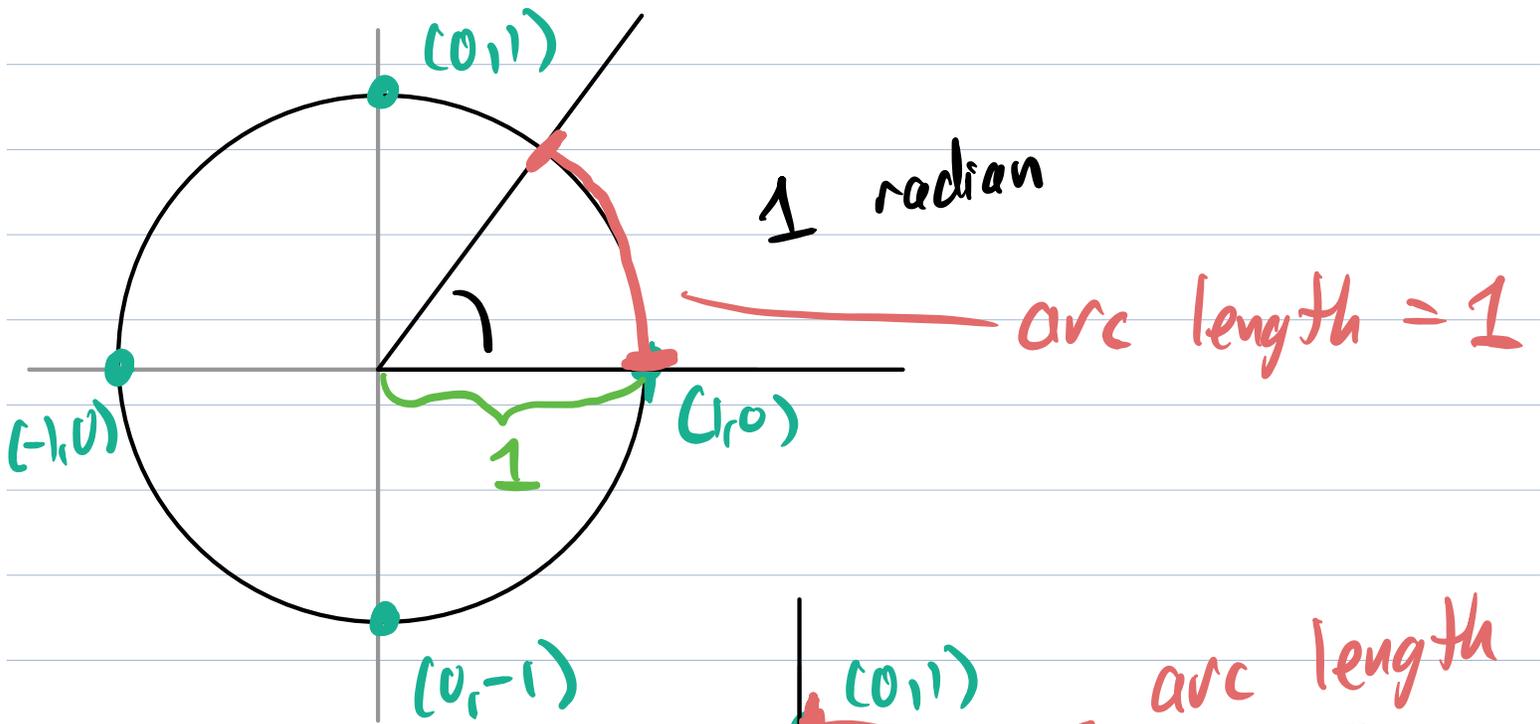
"obtuse" = more than 90°

The point of radians is they correspond to arc length for a "unit circle."

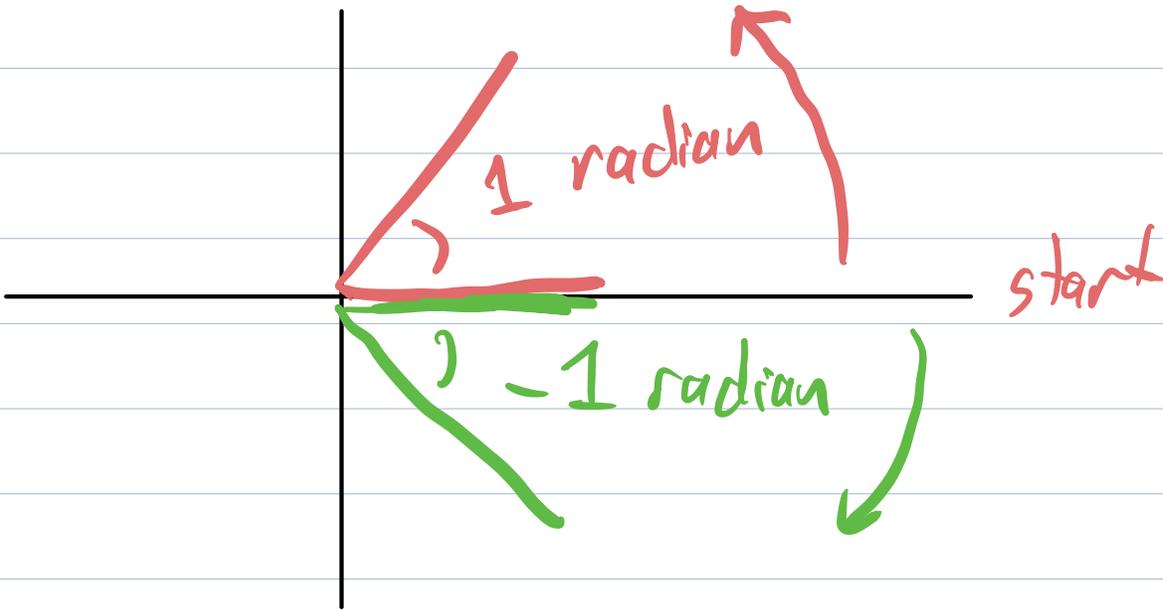
↳ a circle with radius 1

full circle = 2π radians
= circumference 2π

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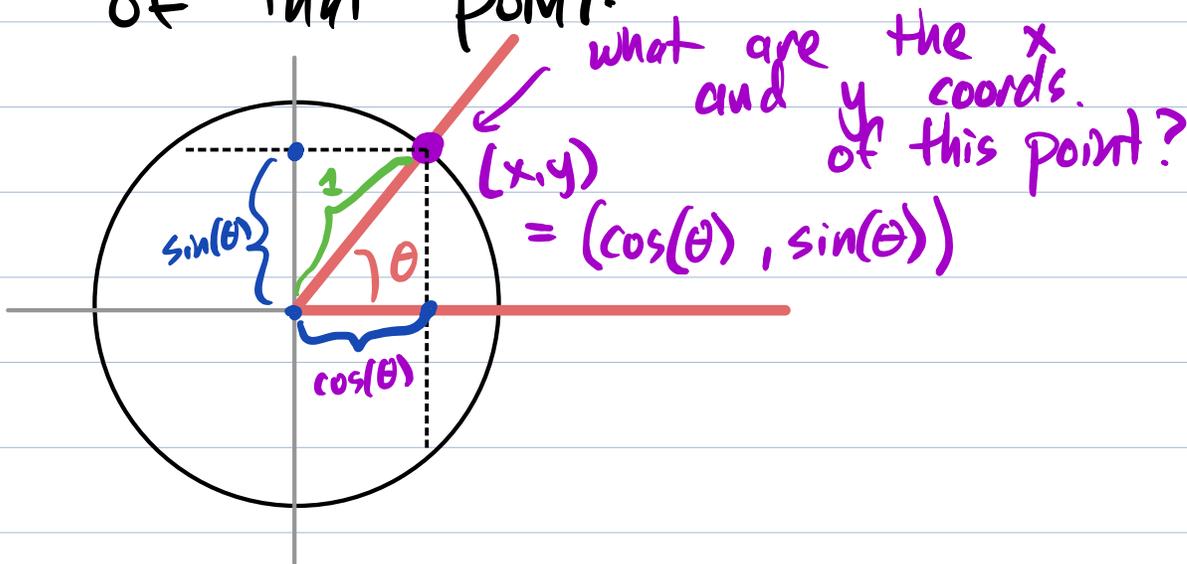


Angles technically a direction. Positive angles go counterclockwise, negative angles go clockwise.



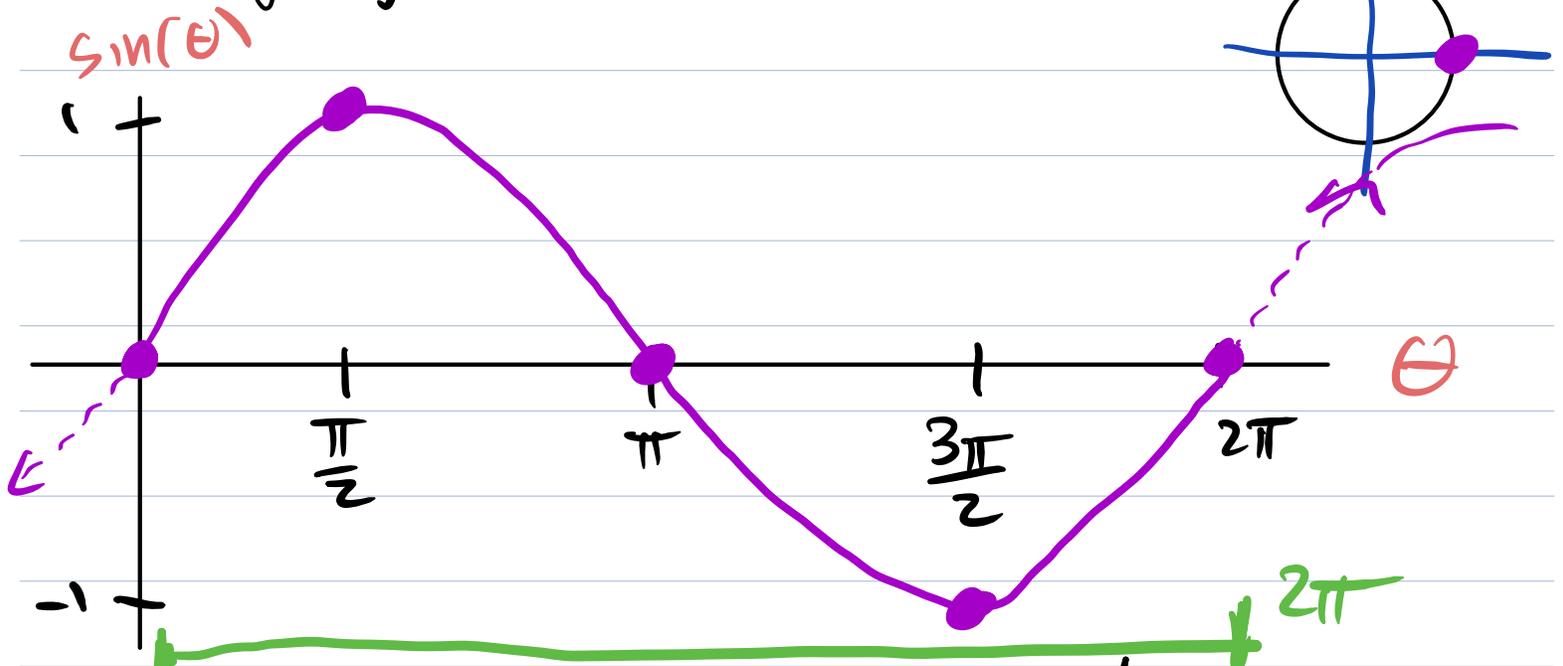
Sine and cosine

As you spin a point around the outside of a circle, the trig functions **cosine** and **sine** tell you the x and y coordinates of that point.



$\sin(\theta)$ - y-coordinate of the point
going around the circle

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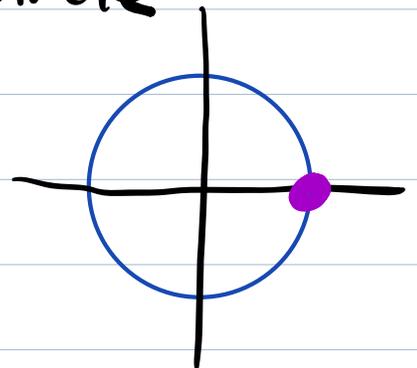
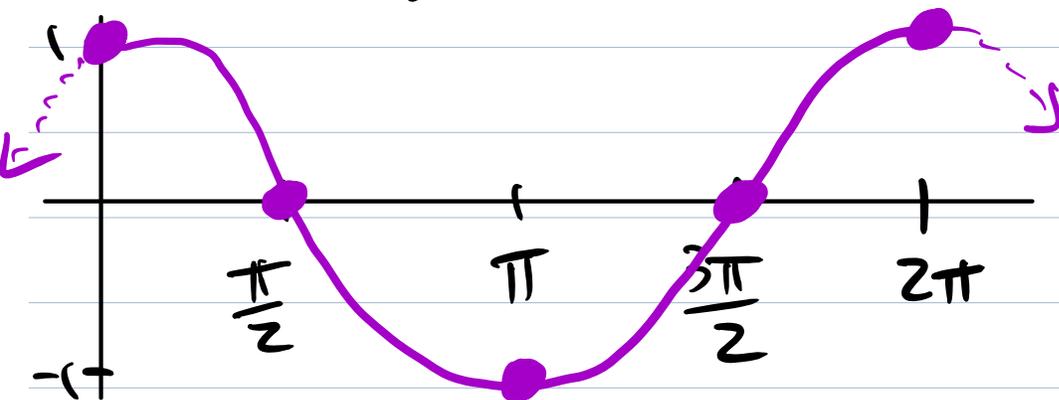
amplitude = distance
from the middle
height to the highest
height

$$A = 1$$

period = how long
until it
repeats

$$P = 2\pi$$

$\cos(\theta)$ - x-coordinate of the point
as it goes around the circle

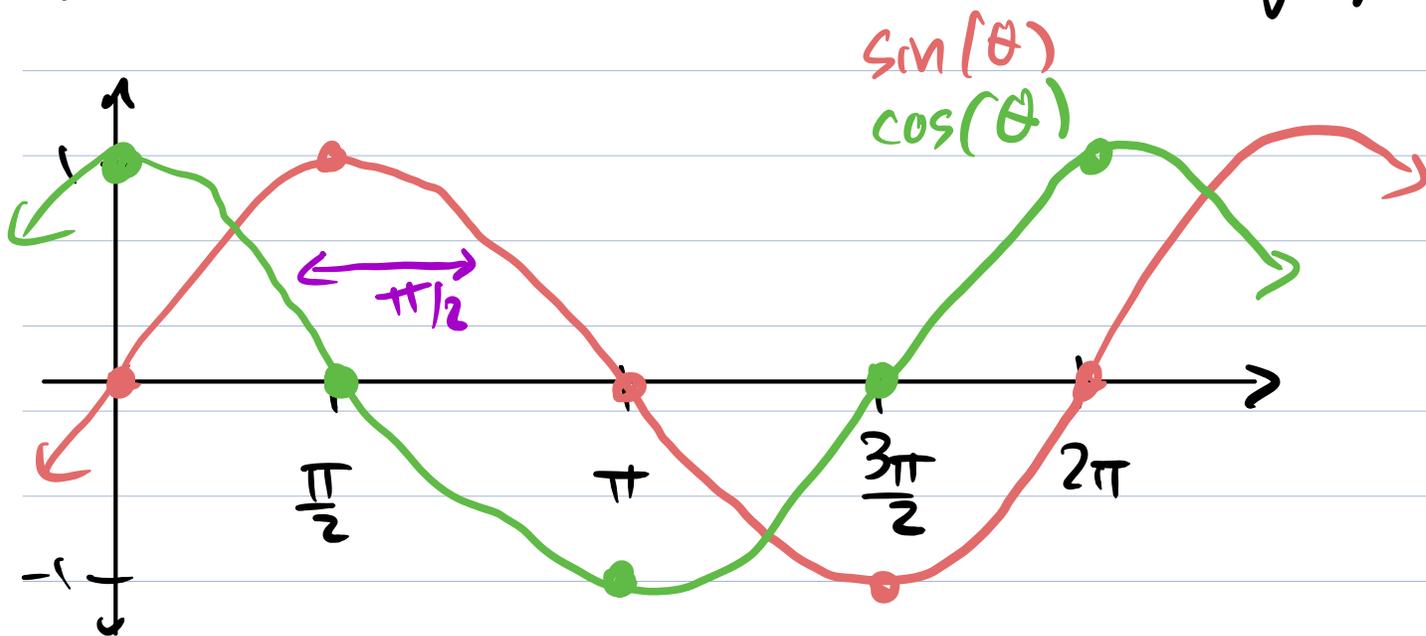


Amplitude = 1

Period = 2π

(6)

sin and cos on the same graph



Section 1.3: $\sin(\theta) = \cos\left(\theta - \frac{\pi}{2}\right)$

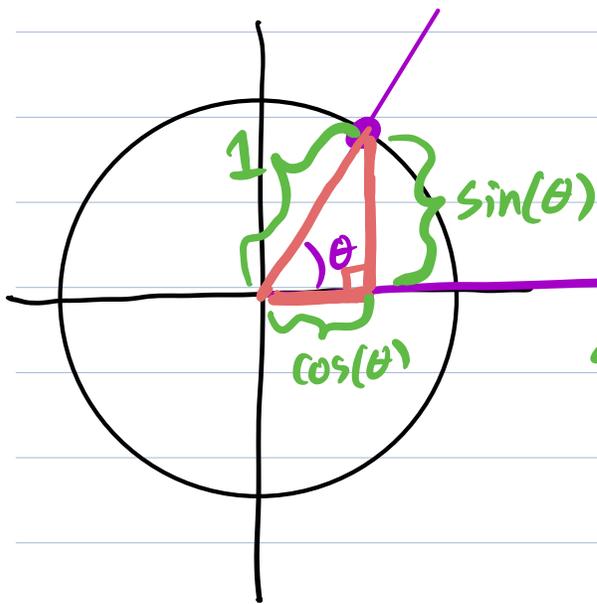
(book has lots more examples)

Notation Instead of $(\sin(\theta))^2$, it is common to write $\sin^2(\theta)$. These are different than $\sin(\theta^2)$.

Fact: For any angle θ :

$$\sin^2(\theta) + \cos^2(\theta) = 1$$

①



$$a^2 + b^2 = c^2$$

$$\sin^2(\theta) + \cos^2(\theta) = 1$$



sin and cos can be transformed like any other function

$$A \cdot \sin(B \cdot \theta)$$

$$|-3| = 3$$

$$|3| = 3$$

$$\text{amplitude} = |A|$$

$$\text{period} = \frac{2\pi}{|B|}$$