Wednesday, Sept. 14 - Fall '22
Lecture # 7

Announcements / Reminders

* WP HW 2 due today 1.2, 1.3 11:59pm
* Q2 tomorrow all 1.2, 1.3, 1.4 sugg HW as of today
* E1 two weeks from today, 9/28

Section 1.4: Logarithmic Functions

$\log_2(5)$ = the # that if you do 2 to the power of it, you get 5.

$2^2 = 4$ $2^{\log_2(5)} = 5$ $2^3 = 8$

$\log_2(5) \approx 2.32$

Properties of Logs - for any base a

(1) $\log_a(x \cdot y) = \log_a(x) + \log_a(y)$

"multiplication inside turns into addition outside"

Why? $2^{x+y} = 2^x \cdot 2^y$

Do "a" to the power of each side.

$$a^{\log_a(x\cdot y)} = a^{\log_a(x)+\log_a(y)}$$

$$x \cdot y = a^{\log_a(x)} \cdot a^{\log_a(y)}$$

$$x \cdot y = x \cdot y \quad \checkmark$$

Not true: ~~$\log_a(x+y) = \log_a(x) \cdot \log_a(y)$~~

(2) $\log_a\left(\frac{x}{y}\right) = \log_a(x) - \log_a(y)$

"Division inside turns into subtraction outside"

Ex: $\log_2\left(\frac{5}{7}\right) = \log_2(5) - \log_2(7)$

$\log_2(0.7\ldots) = \log_2(5) - \log_2(7)$

(3) $\log_a(x^y) = y \cdot \log_a(x)$ $\log_2(5) \approx 2.32$

Ex: $\log_2(25) = \log_2(5^2) = 2 \cdot \log_2(5)$

$\approx 2 \cdot 232 = 4.64\ldots$

$(\log_2(5))^2 \approx (2.32)^2$ not the same thing

$a^{(x \cdot y)} = (a^x)^y$

(4) $\log_a(a^x) = x$ (same "a")

(5) $a^{\log_a(x)} = x$ (same "a")

These are because log is the inverse of exponential.

Group Work:

Simplify: $\log(A^2 \cdot B) - \log\left(\frac{A}{C}\right) + \ln\left(\frac{1}{e^2}\right)$

$= [\log(A^2) + \log(B)] - (\log(A) - \log(C)) + \ln(e^{-2})$

$= 2\log(A) + \log(B) - \log(A) + \log(C) - 2$

$= \log(A) + \log(B) + \log(C) - 2$

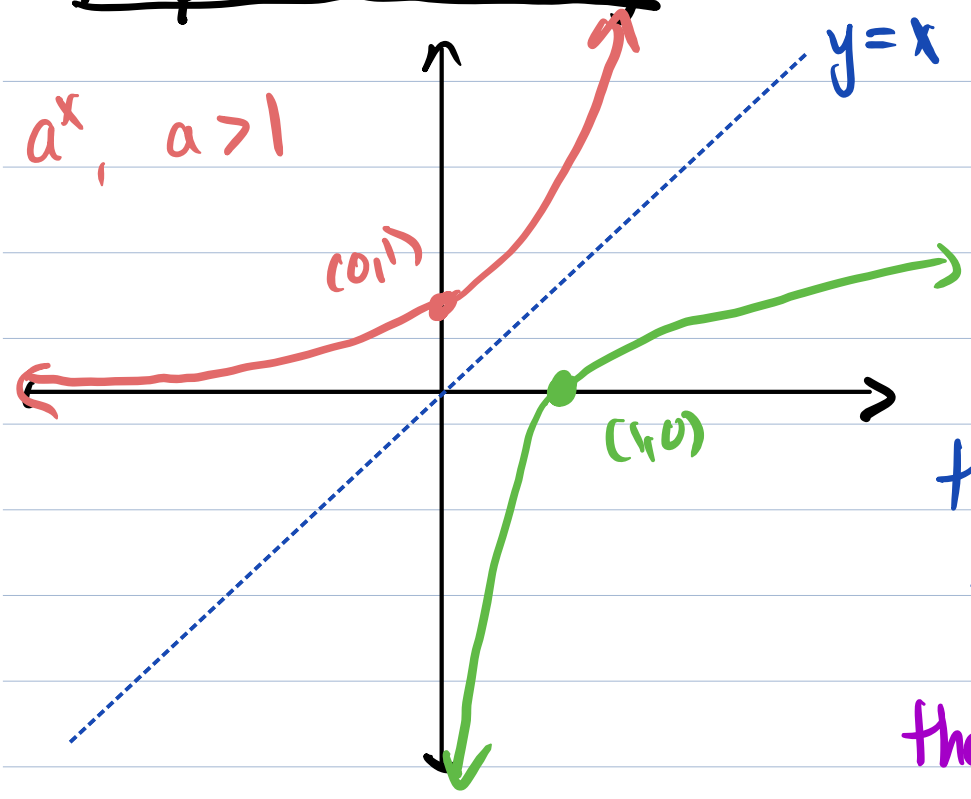
$= \log(ABC) - 2$

$\ln(1) - \ln(e^2)$

$2x - x = x$

$0 - 2 = -2$ (4)

Shape of a log



$2^0 = 1$

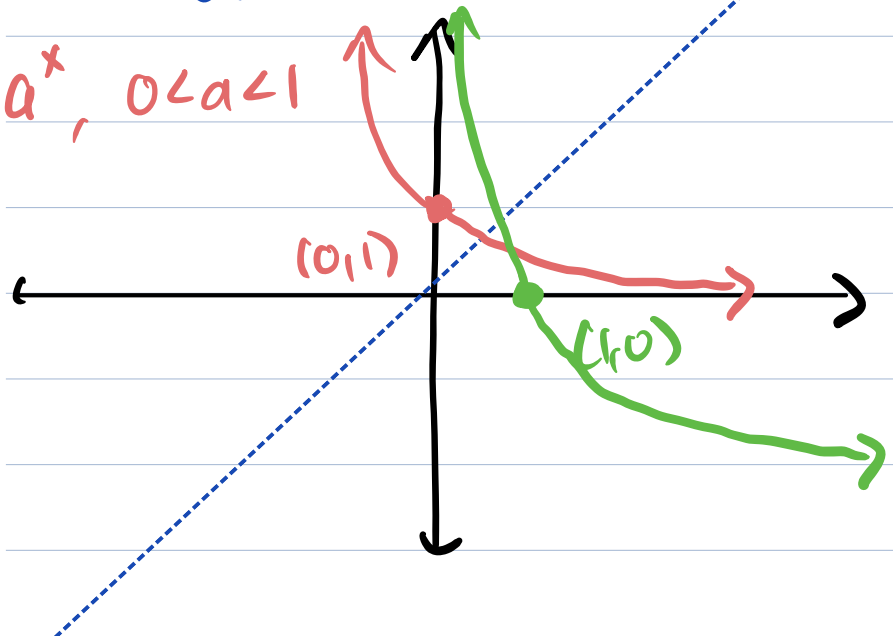
$\log_a(x)$, $a>1$

the domain of $\log_a(x)$:
$(0, \infty)$

the range of $\log_a(x)$:
$(-\infty, \infty)$

$\log(0)$ is undefined

$2^x = 0$? not possible



$\log_a(x)$, $0<a<1$

Ex: Suppose the pop of Burkina Faso is modeled by the function

$$P = 14.235 \cdot (1.029)^t$$

where t is the # of years after 2007 and $P(t)$ is in millions.

When did the population reach 20 million?

We want to solve: $20 = 14.235 \cdot (1.029)^t$.

$$\Rightarrow \frac{20}{14.235} = (1.029)^t$$

$$\Rightarrow \log_{1.029}\left(\frac{20}{14.235}\right) = \log_{1.029}(1.029^t)$$

$$\log_{1.029}\left(\frac{20}{14.235}\right) \approx 11.894 \qquad \log_{1.029}(1.029^t) = t$$

2007 + (11-12 years) → 2018-2019

* Exercises video has a ton of examples