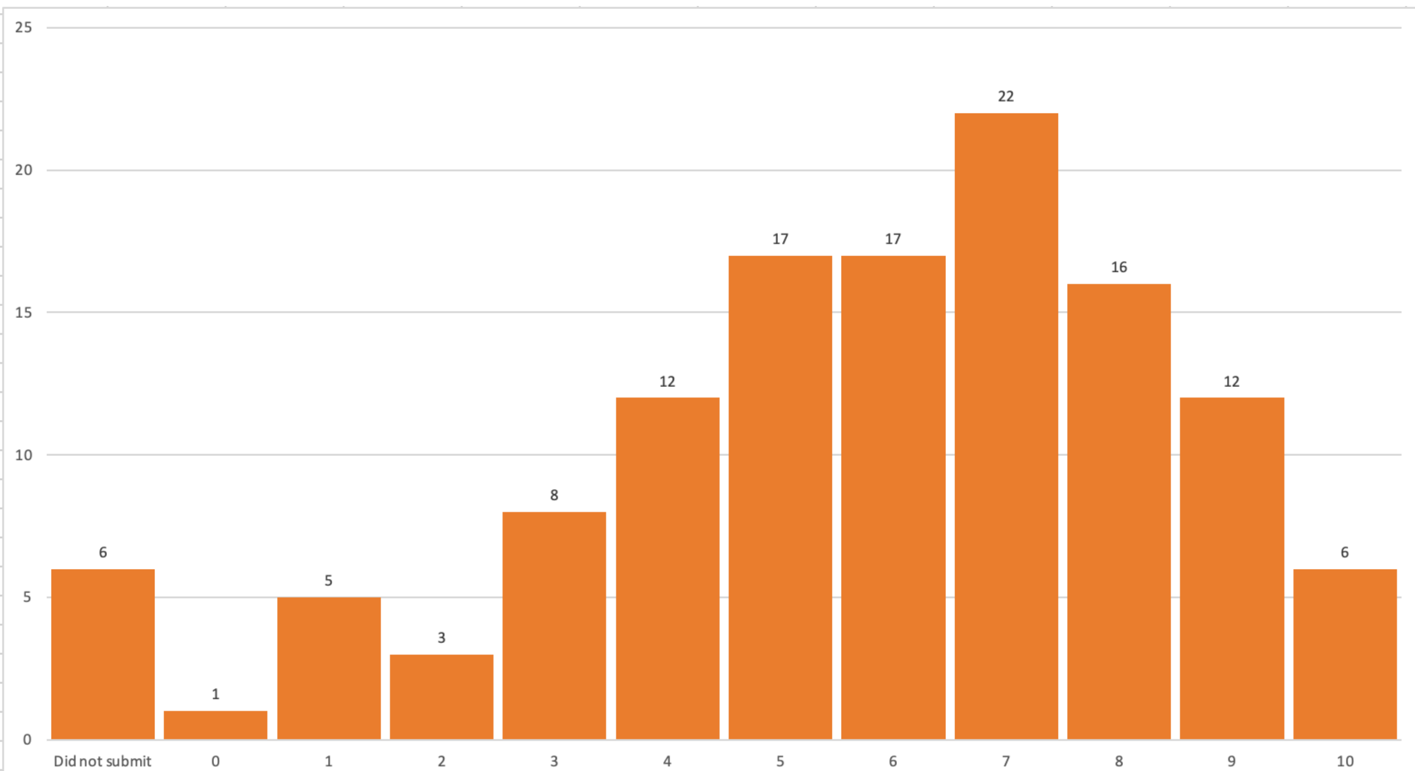
Monday, Sept. 12 - Fall '22
Lecture #6

Announcements / Reminders

* WP HW 2 due Wed 1.2, 1.3 11:59 pm
* Q2 on Thurs, 9/15 1.2, 1.3, 1.4
* Office Hours Tues 12:30 - 1:30 } Cudahy 307
  Fri 8:00 - 9:00 } Help Desk
* Calc Pretest Results:



* Reminder about lecture/exercise videos from Fall 2020.

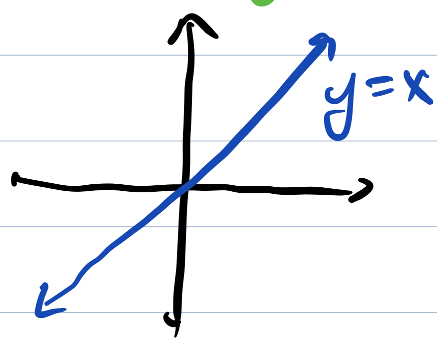
Section 1.3:
Inverse Functions

For a function $f(x)$, the inverse $f^{-1}(x)$ is the function that swaps x and y values of $f(x)$.

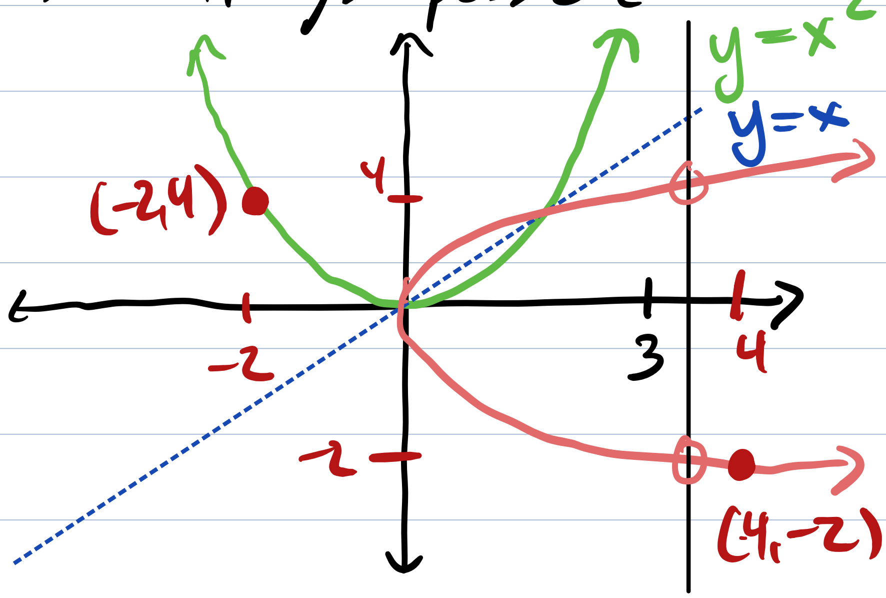
(reflection over the line $y=x$)

$x \to \boxed{f} \to f(x)$

$\leftarrow \boxed{f^{-1}} \leftarrow$



This isn't always possible!

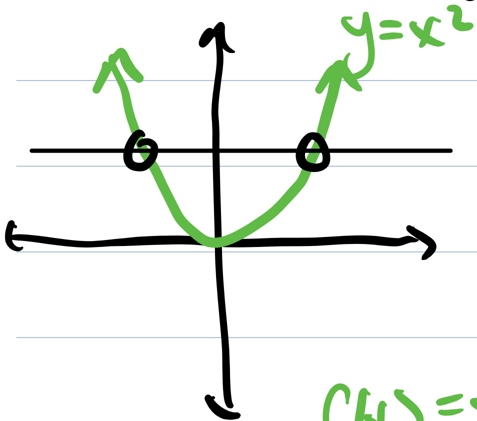


The reflection of the green function is the orange ~~function.~~ thing.

A function cannot have 2 or more outputs for a given input.

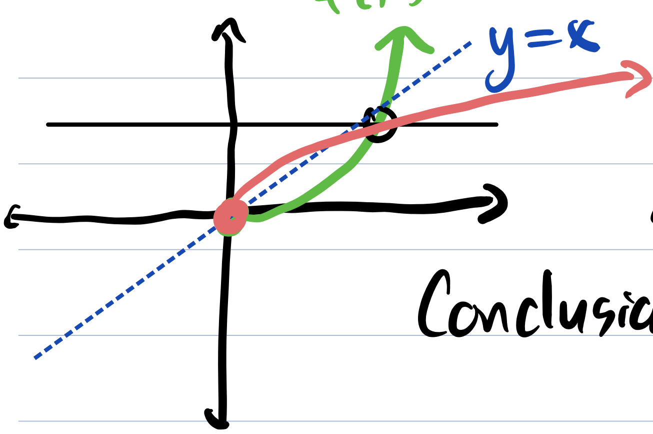
$f(x)=x^2$ does not have an inverse because when we flip it, the result is not even a function.

A function has an inverse if it passes the <u>Horizontal Line Test</u>: There is no horizontal line that passes through the function more than once. (3)



fails the HLT
$f(x)=x^2$ is not invertible.

$f(x)=x^2$ for $x \geq 0$ (Domain: $[0,\infty)$)



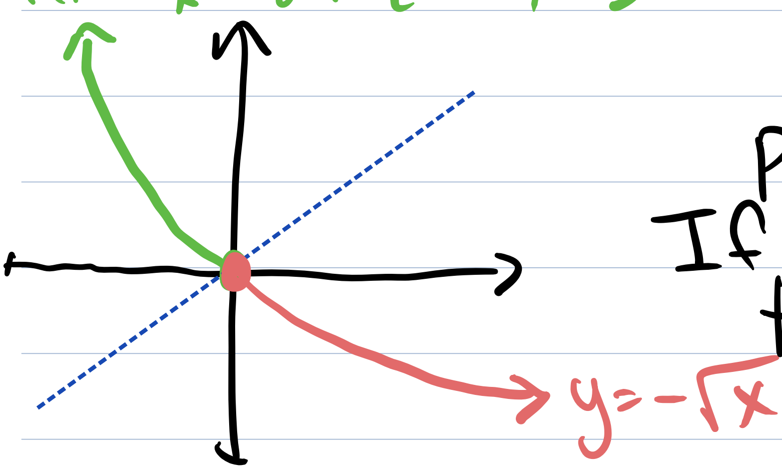
passes the HLT
so it does have an inverse

Conclusion: If $f(x)=x^2$ on the domain $[0,\infty)$, then $f^{-1}(x)=\sqrt{x}$.

($x \geq 0$

$\xrightarrow{x} \boxed{f} \xrightarrow{x^2} \boxed{f^{-1}} \xrightarrow{x}$

$f(x)=x^2$ on $[-\infty,0]$



passes the HLT
If $f(x)=x^2$ on $[-\infty,0]$, then $f^{-1}(x)=-\sqrt{x}$

Calculating the inverse

Let $f(x)$ be an invertible function.

To find a formula for $f^{-1}(x)$, solve for the dependent variable.

chirps/minute

temperature

input: temp
output: c/m

Ex: $C(T) = 4T - 160$

solve for T

temp $\longrightarrow \boxed{} \longrightarrow$ chirps/min

$C = 4T - 160$

$+160 \qquad +160$

$\dfrac{C+160}{4} = \dfrac{4T}{4}$

temp $\longleftarrow \boxed{} \longleftarrow$ chirps/min

input: c/m
output: temp

$T = \dfrac{C}{4} + 40$

$y = 4x - 160$

$x = 4y - 160$

then solve for y

$y = \dfrac{x}{4} + 40$

$C(65) =$ If the temp is $65°$, how many chirps/min?

$4 \cdot 65 - 160 = 100$

$T(100) =$ If there is 100 chirps/min, then what is the temp?

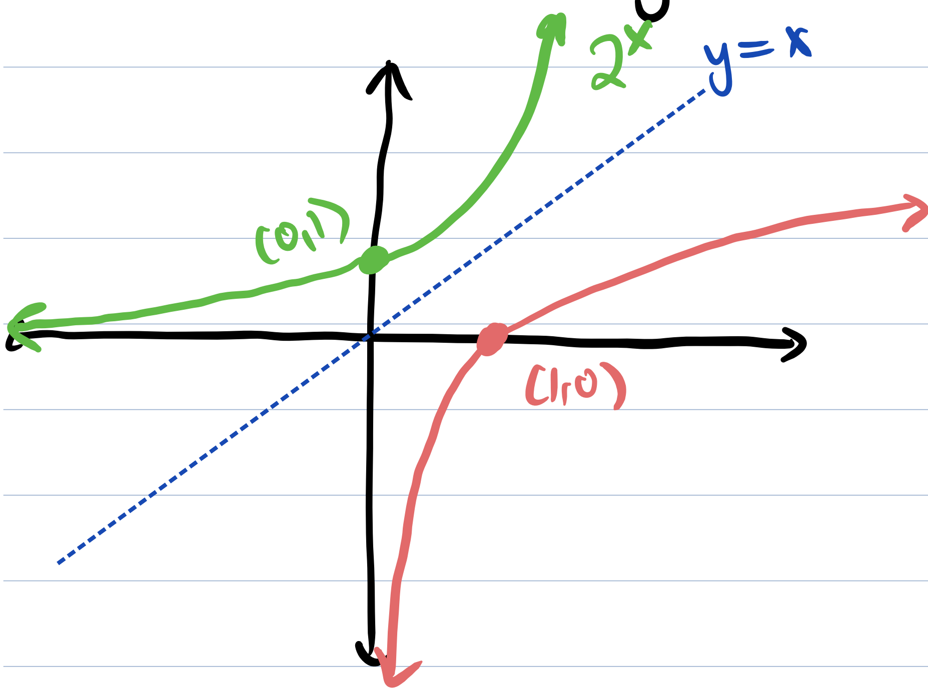
$\dfrac{100}{4} + 40 = 65$

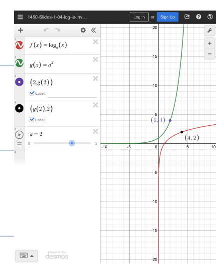
Section 1.4 – Logarithmic Functions

- 1.2 exponential functions
- 1.3 inverse functions

Logarithmic Functions are the <u>inverses</u> of exponential functions.

Since exponential functions have a "base" (a^x), so do logarithmic functions ($\log_a(x)$)





log "undoes" exponentiation

<u>Ex:</u> Solve $5 = 2^x$

Between what two whole #s is x? Between 2 and 3

$2^1 = 2$, $2^2 = 4$, $2^3 = 8$

Take the $\log_2$ of both sides

$$\log_2(5) = \cancel{\log_2}(\cancel{2}^x)$$

log undoes exp

$$\log_2(5) = x$$

$\approx 2.3219\ldots$

This is the # such that if you raise 2 to it, you get 5.

When the base of a log is 10, it's common to just write "log" instead of "$\log_{10}$".

$\approx 2.71\ldots$

When the base is "e", we write "ln" and say "natural log". "ln" = "$\log_e$"

Change of Basis Formula:

You can rewrite $\log_a(x)$ as

$$\frac{\log_c(x)}{\log_c(a)}$$

for any # c.

$$\log_2(5) = \frac{\log_{10}(5)}{\log_{10}(2)} = \frac{\ln(5)}{\ln(2)} = \frac{\log_7(5)}{\log_7(2)}.$$