Wednesday, Sept. 7 — Fall '22
Lecture #4

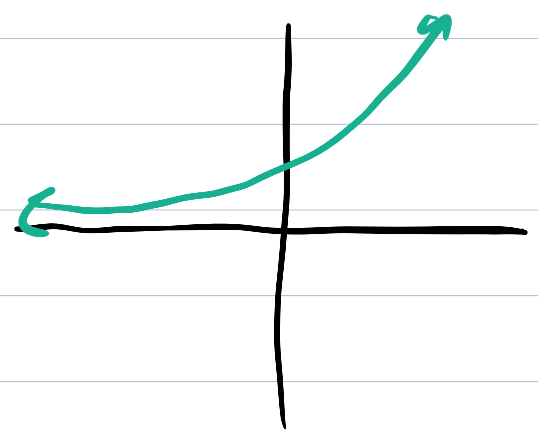
jaypantone.com

Announcements / Reminders

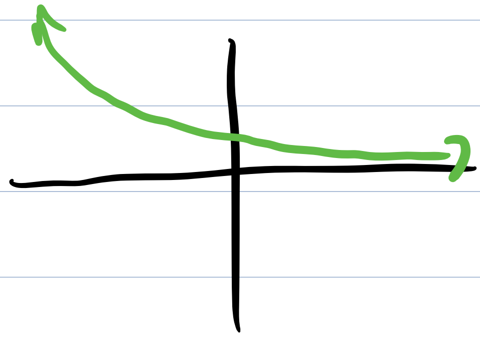
* Turn in calculus pretest by Friday
* WP HW 0 due today! 11:59pm
* Q1 tomorrow covering 1.1
* WP HW 1 due Fri (HW 2 already posted, due next Wed)
* Help Desk! Hours on website.
* If you just joined the class, see me after.

Section 1.2 - Exponential Functions

exponential growth: $P_0 \cdot a^t \quad a > 1$



exponential decay: $P_0 \cdot a^t \quad 0 < a < 1$



24. Each of the functions g, h, k in Table 1.13 is increasing, but each increases in a different way. Which of the graphs in Figure 1.36 best fits each function?

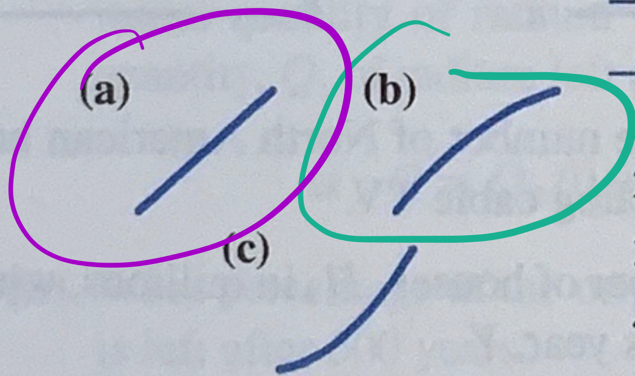


Figure 1.36

Table 1.13

t	$g(t)$	$h(t)$	$k(t)$
1	23	10	2.2
2	24	20	2.5
3	26	29	2.8
4	29	37	3.1
5	33	44	3.4
6	38	50	3.7

0.3
0.3
0.3
0.3

g(t): C
h(t): B
k(t): A

Example:

Your body filters medication from your blood at a rate that depends on the medication. Ampicillin is filtered at a rate of 60% per hour.

Down by 60% = 40% is left.

Suppose you start with 250mg in your blood, and let $f(t)$ be the function for the amount left after t hours.

After 1 hour:

$250 \cdot 0.4 = 100$

$f(t) = 250 \cdot (0.4)^t$

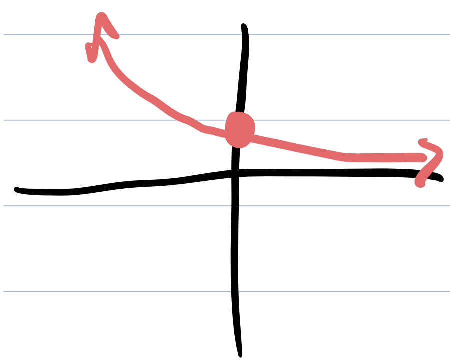
$P_0 \cdot a^t$

$0 < a < 1$

After 2 hours:

$100 \cdot 0.4 = 40$

$250 \cdot 0.4^2 = 40$



Why is it not 0.6?

$250 \cdot 0.6 = 150$ mg

$t=2$ $\quad 250 \cdot 0.6 \cdot 0.6 = 90$ mg

$f(t)$

what does 90 mean?

nothing

Some terminology:

"Doubling Rate": The time it takes an exponentially growing quantity to double.

"Half-Life": the time it takes for an exponentially decaying quantity to reduce by 50%

Base "e"

"e" is a predefined #, kind of like π
$e \approx 2.718\ldots$ (goes on forever without repeating)

You can always rewrite $P_0 \cdot a^t$ to use "e" as the base instead of a.

Ex: $5 \cdot 2^t = 5 \cdot e^{\ln(2) \cdot t} = 5 \cdot \left(\overset{2}{\cancel{e^{\ln(2)}}}\right)^t$

$e^{\ln(2)} = 2$ $\qquad$ $x^{a \cdot b} = (x^a)^b$

$\approx 5 \cdot e^{0.693 \cdot t}$

Works the same for decay:

$$5\cdot\left(\tfrac{1}{3}\right)^{t} = 5\cdot e^{\ln\left(\frac{1}{3}\right)\cdot t}$$

$$\approx 5\cdot e^{-1.099\cdot t}$$

When the base is "e" we call the constant (like 0.693 or -1.099) the "continuous rate"

Topics in 1.2 we didn't cover: Concavity

Section 1.3 – New Functions From Old

Transformations to a function $f(x)$:

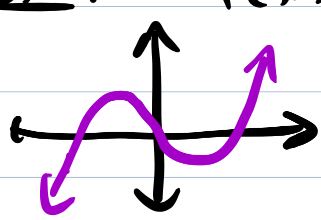
$f(x)+c$ $\quad$ $f(x)-c$

$c\cdot f(x)$ $\quad$ $g(x+2)$

$f(x+c)$ $\quad$ $f(x-c)$

$f(c\cdot x)$ $\quad$ $g(7x)$

Example Function: $f(x) = x^3 - 3x$



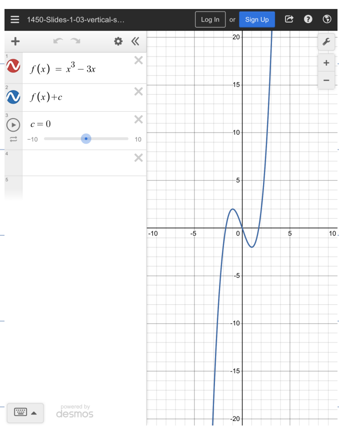
(1) $f(x) \pm c$

Adding a positive # c shifts vertically up by c units.

Adding a negative #c shifts vertically down by c units.

Why? A point on $f(x)$ is $f(2) = 2$.
(2, 2)

Let $g(x) = f(x) + 3$.
$g(2) = f(2) + 3 = 2 + 3 = 5$ (2, 5)



(2) $f(x \pm c)$

Adding a positive # c inside shifts to the left by c units.

Adding a negative #c inside shifts to the right by c units. 7

Why is it backward?

$f(x) = x^3 - 3x$

Define $g(x) = f(x+5)$

$g(0) = f(5)$

(5, 110) on f

(0, 110) on g

The value of g at $x=0$ is the value of f at $x=5$.

g pulls its values from 5 to the right, so they move 5 to the left.