MATH 2100 / 2350 – HOMEWORK 6 (LAST ONE!)

Fall 2020

due **Monday**, **November 23**, on D2L, by the beginning of class

$\approx$ Sections 3.3, 4.1, 4.2, 4.3

This homework assignment was written in LaTeX. You can find the source code on the course website.

Instructions: This assignment is due on D2L at the *beginning* of class. It must be typed in Latex (other formats such as Word are not acceptable). **You must submit the .pdf file, but you do not have to submit the .tex file unless I ask for it** Any pictures can be drawn by hand and added to the Latex file with the "\includegraphics" command (see how I do it in this document). Please write the questions in the correct order. Explain all reasoning.

Note: *You no longer need to include your scratch work with each proof.*

1. Prove or disprove: For any two sets A and B: $\mathcal{P}(A) \cup \mathcal{P}(B) \subseteq \mathcal{P}(A \cup B)$.
2. Prove the following set inequality:
$$(\{6k+1 : k \in \mathbb{Z}\} \cup \{6m-1 : m \in \mathbb{Z}\}) \subseteq \{2n+1 : n \in \mathbb{Z}\}.$$
3. Draw the two-sided arrow diagram for the function $f : \mathcal{P}(\{4,5,6\}) \to \mathcal{P}(\{1,2,3\})$ defined by
$$f(S) = \{x-3 : x \in S\} \smallsetminus \{2\}.$$
4. Prove that the function $h : \mathbb{N} \to \mathbb{N}$ defined by $h(n) =$ [the sum of the digits in n (in base 10)] is surjective. Prove that it's not injective.
5. Let $c : \mathcal{P}(\{x,y,z\}) \to \mathcal{P}(\{x,y,z\})$ be the function with the rule $c(A) = \{x,y,z\} \smallsetminus A$, and let $n : \mathcal{P}(\{x,y,z\}) \to \{0,1,2,3\}$ be the function such that $n(A)$ is the number of elements in the set A. Which composition makes sense, $c \circ n$ or $n \circ c$? For the one that is defined, give the domain, codomain, range, and draw the two-sided arrow diagram.
6. Let $A = \{0,1,2,3\}$ and let $B = \{000, 001, 010, 011, 100, 101, 110, 111\}$ be the set of binary strings with three digits. Define $g : B \to A$ by $g(s) =$ [the number of 1s in s]. Draw the arrow diagram for the function. Determine whether or not it's injective, surjective, and bijective. Make sure to justify your answers (either with the arrow diagram, or a formal proof).