MATH 2100 / 2350 – HOMEWORK 4

Fall 2020

due Wednesday, **October 28**, on D2L, by the beginning of class

Sections 2.1, 2.2, 2.3, 2.4

This homework assignment was written in LaTeX. You can find the source code on the course website.

Instructions: This assignment is due on D2L at the *beginning* of class. It must be typed in Latex (other formats such as Word are not acceptable). **You must submit the .pdf file, but you do not have to submit the .tex file unless I ask for it** Any pictures can be drawn by hand and added to the Latex file with the "\includegraphics" command (see how I do it in this document). Please write the questions in the correct order. Explain all reasoning.

Note: You can write a proof in Latex with the commands

```
\begin{proof}
    proof goes here
\end{proof}
```

1. Decide if the following statement is true or false. If it's true, prove it. If it's false, provide a counterexample.

   If A, B, and C are integers and if A divides B and B divides C, then A divides C.

2. Decide if the following statement is true or false. If it's true, prove it. If it's false, provide a counterexample.

   If A, B, and C are integers and if A divides B and A divides C, then $A \cdot B$ divides C.

3. Decide if the following statement is true or false. If it's true, prove it. If it's false, provide a counterexample.

   If n is a positive even integer, then $3^n + 1$ is divisible by 5.

4. Decide if the following statement is true or false. If it's true, prove it. If it's false, provide a counterexample.

   If n is a positive even integer, then $n^3 + 2n$ is divisible by 4.

5. Decide if the following statement is true or false. If it's true, prove it. If it's false, provide a counterexample.

   If m is a positive odd integer, then $m^2 - 1$ is divisible by 8.

6. Prove that if 3 divides $4^{n-1} - 1$ then 3 divides $4^n - 1$.

7. Prove that for all positive integers n,

$$\sum_{k=0}^{n} (k \cdot k!) = (n+1)! - 1.$$

8. Prove that for all positive integers $n \geq 2$, the number $2^{3n} - 1$ is not prime.

9. Prove that for all positive integers $n \geq 4$,

$$n! > 2^n.$$