# MATH 2100 / 2350 – HOMEWORK 2

Fall 2020

due Wednesday, **September 23**, on D2L, by the beginning of class

Sections 1.3, 1.4, 1.5

*This homework assignment was written in LaTeX. You can find the source code on the course website.*

**Instructions:** This assignment is due on D2L at the *beginning* of class. It must be typed in Latex (other formats such as Word are not acceptable). **You must submit the .pdf file, but you do not have to submit the .tex file unless I ask for it** Any pictures can be drawn by hand and added to the Latex file with the "\includegraphics" command (see how I do it in this document). Please write the questions in the correct order. Explain all reasoning.

**Mathematical Writing:** An important component of this course is learning how to write mathematics correctly and concisely. Your goal should always be the convince the reader that you are correct! That means explaining your thinking and each step in your solution. We will talk more about this when we cover formal proofs in a few weeks, but for now I expect you to do the following: explain your reasoning, don't leave out steps, and use full sentences with correct spelling and grammar (including your use of math symbols). For example, don't write "$3 \in S \implies 3 \notin \overline{S}$"; instead, write "Since $3 \in S$, it follows that $3 \notin \overline{S}$".

1. Write each of the sentences below using predicates and quantifiers. Each one tells you how many quantifiers your answer should have. Make sure you define each set clearly (like we did in class).
   (a) Every exam has at least one typo. — 2 quantifiers
   (b) Some cats do not like having their belly rubbed. — 1 quantifier
   (c) Nobody likes prunes. — 1 quantifier
   (d) Some books have characters that nobody likes. — 3 quantifiers
2. For each of the four sentences above, write the negation. (Note: It will be very helpful to start with your previous answer that you put in predicate-quantifier form!)
3. Give your own example, different from the ones in class, of a predicate $P(x, y)$ such that $\forall x, \exists y, P(x, y)$ and $\exists y, \forall x, P(x, y)$ mean different things. Explain what each version means. (Each student in the class should have a different answer.)
4. Let $Q(a, b) =$ "$3a + b = 5$" and assume for the rest of this question that $a$ and $b$ are always rational numbers (that means they are either 0 or any fraction $p/q$ where $p$ and $q$ are positive or negative whole numbers). Which of the following are true? Justify your answers, stating explicitly whether you're justifying by giving a single example, or by stating something for all cases.
   (a) $Q(1, 2)$
   (b) $\exists x, Q(x, 0)$
   (c) $\exists x, ((x \neq 0) \wedge Q(x, 0))$
   (d) $\forall x, Q(0, x)$
   (e) $\exists y, Q(y, y)$

(f) $\forall x, \exists y, Q(x, y)$

(g) $\exists x, \forall y, Q(x, y)$

5. For each statement below, translate from English to Math, using predicates, quantifiers, and implications where necessary. *The actual statements might be true or false, so you don't need to prove any of these!*

   (a) For all real numbers $r$, if $r^2 > 0$ then $r > 0$.

   (b) There exists a real number $m$, such that for all rational numbers $q$, $m = q^2$.

   (c) For all natural numbers $n$ and $k$, if $n$ is a multiple of $k$, then $n^2$ is a multiple of $k^2$.

6. Write the negation of each implication in English. It may help to translate to math first.

   (a) If it is snowing, then it is cold outside.

   (b) If it is cold outside, then it is snowing.

   (c) If I have three cats and a dog, then I have too many pets!

   (d) If I go to sleep too late or I eat ice cream for breakfast, then I don't feel good.

7. Use a truth table to determine if the two propositions are logically equivalent:

$$q \to ((p \to r) \land (r \to p)) \qquad\qquad q \land r$$

8. For each part below, devise an implication that satisfies the conditions of that part (not all parts at once), or if it's not possible, explain why not. Your answer should be different from any of the examples we did in class.

   (a) an implication that is false, but its converse is true

   (b) an implication that is false, but its contrapositive is true

   (c) an implication that is false, and its converse is false