MATH 2100 / 2350 – HOMEWORK 1

Fall 2020

due ~~Wednesday, **September 9**~~ Friday, **September 11**, on D2L, by the beginning of class

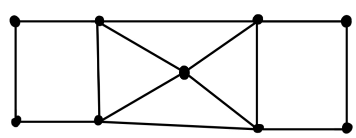
Sections 7.1, 1.3, 1.4

This homework assignment was written in LaTeX. You can find the source code on the course website.

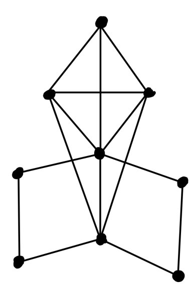
Instructions: This assignment is due on D2L at the *beginning* of class. It must be typed in Latex (other formats such as Word are not acceptable). **You must submit both the .tex file and the .pdf file!** Any pictures can be drawn by hand and added to the Latex file with the "\includegraphics" command (see how I do it in this document). Please write the questions in the correct order. Explain all reasoning.

Mathematical Writing: An important component of this course is learning how to write mathematics correctly and concisely. Your goal should always be the convince the reader that you are correct! That means explaining your thinking and each step in your solution. We will talk more about this when we cover formal proofs in a few weeks, but for now I expect you to do the following: explain your reasoning, don't leave out steps, and use full sentences with correct spelling and grammar (including your use of math symbols). For example, don't write "$3 \in S \implies 3 \notin \overline{S}$"; instead, write "Since $3 \in S$, it follows that $3 \notin \overline{S}$".

1. For the graph below, label the degree of every vertex. Find an Eulerian path (if one exists), or explain why there can't be one (if one doesn't exist). Do the same thing for an Eulerian circuit. *When giving a path or circuit, be sure to find a way to present it on paper that makes sense!*



2. For the graph below, label the degree of every vertex. Find an Eulerian path (if one exists), or explain why there can't be one (if one doesn't exist). Do the same thing for an Eulerian circuit. *When giving a path or circuit, be sure to find a way to present it on paper that makes sense!*



3. (a) Draw a graph that *requires* five different colors in order to assign a color to each vertex with the property that no two vertices connected by an edge have the same color.

(b) Draw a graph that *requires* six different colors in order to assign a color to each vertex with the property that no two vertices connected by an edge have the same color.

(c) Explain how for any positive whole number n, you could draw a graph that *requires* n different colors in order to assign a color to each vertex with the property that no two vertices connected by an edge have the same color.

4. Determine whether the following logical equivalence is true by drawing a truth table. Make sure that you explain how you are using your truth table to draw your conclusion.

$$\neg((r \wedge p) \vee \neg q) \equiv \neg(r \wedge p) \wedge q.$$

5. You come across three inhabitants of an island, Ralph, Sharon, and Tito. Each of them either always lies or always tells the truth. Ralph says "None of us are liars," Sharon says "None of us tell the truth," and Tito says "I'm telling the truth!" What are the possible combinations of whether each person is lying or telling the truth? (There could be no combinations, one combination, or more than one combination!)

6. ~~Write each of the sentences below using predicates and quantifiers. Each one tells you how many quantifiers your answer should have. Make sure you define each set clearly (like we did in class).~~

   (a) ~~Every exam has at least one typo. — 2 quantifiers~~

   (b) ~~Some cats do not like having their belly rubbed. — 1 quantifier~~

   (c) ~~Nobody likes prunes. — 1 quantifier~~

   (d) ~~Some books have characters that nobody likes. — 3 quantifiers~~

7. ~~For each of the four sentences above, write the negation. (Note: It will be very helpful to start with your previous answer that you put in predicate-quantifier form!)~~

8. ~~Give your own example, different from the ones in class, of a predicate $P(x, y)$ such that $\forall x, \exists y, P(x, y)$ and $\exists y, \forall x, P(x, y)$ mean different things. Explain what each version means. (Each student in the class should have a different answer.)~~

9. ~~Let $Q(a, b) =$ "$3a + b = 5a$" and assume for the rest of this question that a and b are always rational numbers (that means they are either 0 or any fraction p/q where p and q are positive or negative whole numbers). Which of the following are true? Justify your answers, stating explicitly whether you're justifying by giving a single example, or by stating something for all cases.~~

   (a) ~~$Q(1, 2)$~~

   (b) ~~$\exists x, Q(x, 0)$~~

   (c) ~~$\exists x, ((x \neq 0) \wedge Q(x, 0))$~~

   (d) ~~$\forall x, Q(0, x)$~~

   (e) ~~$\exists y, Q(y, y)$~~

   (f) ~~$\forall x, \exists y, Q(x, y)$~~

   (g) ~~$\exists x, \forall y, Q(x, y)$~~