

MATH 2100 / 2350 – HOMEWORK 5

Fall 2019

due Wednesday, **November 13**, at the beginning of class

Sections 2.5, 3.3, 4.1

This homework assignment was written in \LaTeX . You can find the source code on the course website.

Instructions: This assignment is due at the *beginning* of class. **Staple your work** together (do not just fold over the corner). Please write the questions in the correct order. If I cannot read your handwriting, you won't receive credit. Explain all reasoning.

1. Prove that if any five points other than $(0,0)$ are placed on the coordinate plane, then there are two points, call them A and B , such that the angle formed by the rays from $(0,0)$ to A and from $(0,0)$ to B is acute.
2. Use a proof by contradiction to prove that an even perfect square cannot have the form $4k + 2$.
3. Prove that if $a + b + c \geq 35$, then either $a \geq 10$, $b \geq 12$, or $c \geq 13$.
4. Prove or disprove: For any two sets A and B : $\mathcal{P}(A) \cup \mathcal{P}(B) \subseteq \mathcal{P}(A \cup B)$.
5. Prove or disprove: For any three sets A , B , and C : $(A \cup B) \times C = (A \times C) \cup (B \times C)$.
6. Prove the following set inequality:

$$(\{n^2 - 1 : n \in \mathbb{Z}\} \cap \{2k : k \in \mathbb{N}\}) \subseteq \{4m : m \in \mathbb{Z}\}.$$

7. Prove the following set inequality:

$$(\{6k + 1 : k \in \mathbb{Z}\} \cup \{6m - 1 : m \in \mathbb{Z}\}) \subseteq \{2n + 1 : n \in \mathbb{Z}\}.$$

8. Draw the one-sided and two-sided arrow diagrams for the function $f : \mathcal{P}(\{4,5,6\}) \rightarrow \mathcal{P}(\{1,2,3\})$ defined by

$$f(S) = \{x - 3 : x \in S\} \setminus \{2\}.$$