MATH 2100 / 2350 – HOMEWORK 4

Fall 2019

due Wednesday, **October 30**, at the beginning of class

Sections 2.2, 2.3, 2.4, half of 2.5

This homework assignment was written in LaTeX. You can find the source code on the course website.

Instructions: This assignment is due at the *beginning* of class. **Staple your work** together (do not just fold over the corner). Please write the questions in the correct order. If I cannot read your handwriting, you won't receive credit. Explain all reasoning.

1. Decide if the following statement is true. If it is, prove it. If it's not, provide a counterexample.

   For integers x, y, and z, if x divides y and x divides z, then x^2 divides yz.

2. Decide if the following statement is true. If it is, prove it. If it's not, provide a counterexample.

   For integers x, y, and z, if x divides z and y divides z, then xy divides z^2.

3. Prove that for all positive integers n,
$$1^3 + 2^3 + \cdots + n^3 = (1 + 2 + \cdots + n)^2.$$

4. Prove that for all positive integers n,
$$\sum_{k=0}^{n} (k \cdot k!) = (n+1)! - 1.$$

5. Prove that for all positive integers $n \geq 2$, the number $2^{3n} - 1$ is not prime.

6. Prove that for all positive integers $n \geq 4$,
$$n! > 2^n.$$

7. Prove that at a completely full Milwaukee Bucks game at the Fiserv Forum, there *must* be at least two people that have both the same birthday *and* the same first initial. (Note: you will have to look up the capacity of the arena!)

8. Use the pigeonhole principle to prove that given any five integers, there will be two that have a sum or difference divisible by 7.