

# MATH 2100 / 2350 – HOMEWORK 3

Fall 2019

due Wednesday, **October 16**, at the beginning of class

Sections 2.1, 2.2

*This homework assignment was written in  $\LaTeX$ . You can find the source code on the course website.*

**Instructions:** This assignment is due at the *beginning* of class. **Staple your work** together (do not just fold over the corner). Please write the questions in the correct order. If I cannot read your handwriting, you won't receive credit. Explain all reasoning.

1. Read both of the handouts from class (they're also posted on our website). Write yourself a page of bullet points for things to remember when writing proofs. Keep it somewhere safe, and bring it out every time you write proofs for the next few weeks so you can skim it over and make sure you've followed your bullet points.

**For this question, turn in a copy of your sheet. Keep an original copy for yourself.**

2. Prove that if  $n$  is an integer and  $n^2$  is odd, then  $n$  is odd.
3. Prove that if  $n$  is an integer and 3 divides  $4^{n-1} - 1$ , then 3 divides  $4^n - 1$ .
4. Prove that if  $n$  is an integer and  $n^2$  is divisible by 5, then  $n$  is divisible by 5. Is the same fact true if 5 is replaced by 6? Is it true if 5 is replaced by 4?
5. Prove that if  $a$ ,  $b$ , and  $c$  are integers and if  $a$  divides  $b$  and  $b$  divides  $c$ , then  $a$  divides  $c$ .
6. Decide if the following statement is true. If it is, prove it. If it's not, provide a counterexample.

If  $n$  is a positive even integer, then  $3^n + 1$  is divisible by 5.

7. Decide if the following statement is true. If it is, prove it. If it's not, provide a counterexample.

If  $n$  is a positive even integer and  $n \geq 4$ , then  $2^n - 1$  is not prime.

8. Prove that if  $n$  is an even integer, then  $4(n + 1) + 3$  is odd.