

MATH 2100 / 2350 – HOMEWORK 2

Fall 2019

due Wednesday, **October 2**, at the beginning of class

Sections 1.4, 1.5, 3.1, 3.2

This homework assignment was written in L^AT_EX. You can find the source code on the course website.

Instructions: This assignment is due at the *beginning* of class. **Staple your work** together (do not just fold over the corner). Please write the questions in the correct order. If I cannot read your handwriting, you won't receive credit. Explain all reasoning.

Mathematical Writing: An important component of this course is learning how to write mathematics correctly and concisely. Your goal should always be to convince the reader that you are correct! That means explaining your thinking and each step in your solution. We will talk more about this when we cover formal proofs in a few weeks, but for now I expect you to do the following: explain your reasoning, don't leave out steps, and use full sentences with correct spelling and grammar (including your use of math symbols). For example, don't write " $3 \in S \implies 3 \notin \bar{S}$ "; instead, write "Since $3 \in S$, it follows that $3 \notin \bar{S}$ ".

1. Negate the following statements.
 - (a) If it is raining, then there is lightning.
 - (b) If n is a natural number, then either n is even or n is odd.
2. Use a truth table to determine whether the two statements are equivalent.

$$(p \vee q) \rightarrow (p \wedge q) \qquad p \rightarrow q$$

3. Form a predicate and a quantified statement that represents the following sentence: "Every university has a dorm that is at least 20 years old."
4. Negate the statement "There is a car that everyone wants to buy."
5. Use Venn Diagrams to determine whether the equation below is true:

$$(A \cup B) \setminus (A \cap C) = B \cup (A \setminus C).$$

6. List 5 elements of each of the following sets, unless there are fewer than 5 elements (in which case, list them all and justify how you know you've listed all of them).
 - (a) $\{x \in \mathbb{R} : x \notin \mathbb{N} \text{ and } x^2 \in \mathbb{N}\}$
 - (b) $\{S \subseteq \mathbb{N} : \text{the sum of the elements in } S \text{ is less than } 3\}$
 - (c) $\{z \in \mathbb{N} : z = 5k + 2 \text{ for some } k \in \mathbb{Z}\}$
 - (d) $\{r \in \mathbb{Z} : r = 2k \text{ for some } k \in \mathbb{Z} \text{ and } r = 2\ell + 1 \text{ for some } \ell \in \mathbb{Z}\}$

7. Write each of the following sets in set-builder notation.

- (a) The set A of real numbers that are not rational numbers.

- (b) The set B of rational numbers whose numerator is 1 and whose denominator is a prime number.
 - (c) The set C of pairs of real numbers (r_1, r_2) that add up to a natural number.
 - (d) The set D of subsets of the real numbers whose size is 10 or less.
8. Determine whether the statement below is true or false. If true, give a few sentences of justification (a formal proof is not necessary). If false, give specific examples of sets that make the statement false.

For all sets $A, B,$ and $C:$ if $A \subseteq B$ and $A \subseteq C,$ then $A \subseteq B \cup C.$

9. Determine whether the statement below is true or false. If true, give a few sentences of justification (a formal proof is not necessary). If false, give specific examples of sets that make the statement false.

For all sets A and $B:$ $(A \cup B) \times (A \setminus B) = (A \times A) \setminus (B \times B).$

10. Determine whether the statement below is true or false. If true, give a few sentences of justification (a formal proof is not necessary). If false, give specific examples of sets that make the statement false.

For all sets A and $B:$ if $A \subseteq B$ then $A \subseteq \mathcal{P}(B).$