MATH 2100 / 2350 – HOMEWORK 1

Fall 2019

due Wednesday, **September 18**, at the beginning of class

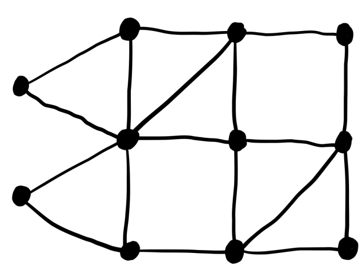
Sections 7.1, 1.3, 1.4, 1.5

This homework assignment was written in LaTeX. You can find the source code on the course website.

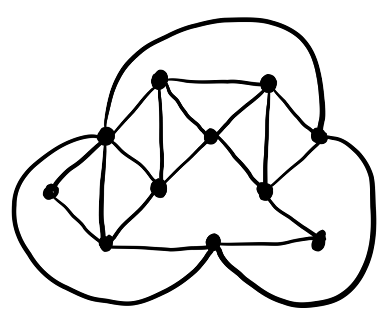
Instructions: This assignment is due at the *beginning* of class. **Staple your work** together (do not just fold over the corner). Please write the questions in the correct order. If I cannot read your handwriting, you won't receive credit. Explain all reasoning.

Mathematical Writing: An important component of this course is learning how to write mathematics correctly and concisely. Your goal should always be the convince the reader that you are correct! That means explaining your thinking and each step in your solution. We will talk more about this when we cover formal proofs in a few weeks, but for now I expect you to do the following: explain your reasoning, don't leave out steps, and use full sentences with correct spelling and grammar (including your use of math symbols). For example, don't write "$3 \in S \implies 3 \notin \overline{S}$"; instead, write "Since $3 \in S$, it follows that $3 \notin \overline{S}$".

1. For the graph below, find an Eulerian path (if one exists), or explain why there can't be one (if one doesn't exist). Do the same thing for an Eulerian circuit. *When giving a path or circuit, be sure to find a way to present it on paper that makes sense!*



2. For the graph below, find an Eulerian path (if one exists), or explain why there can't be one (if one doesn't exist). Do the same thing for an Eulerian circuit. *When giving a path or circuit, be sure to find a way to present it on paper that makes sense!*



3. You come across three inhabitants of an island, Xavier, Yvette, and Zane. Xavier says "The number of us that are lying is odd," Yvette says "The number of us that are lying is even," and Zane says "The number of us that are lying is positive." What are the possible combinations of whether each person is lying or telling the truth? (There could be none, one, or more than one!)

4. Determine whether the following logical equivalence is true:

$$(p \vee r) \wedge (p \vee q) \wedge (\neg q \vee \neg r) \equiv p \wedge \neg(q \wedge r).$$

5. This is a different type of island-logic-question. You might need another method to answer it. Every inhabitant of an island either always tells the truth or always lies. Each of them knows what type of inhabitant they all are, but you as an outsider do not. You come to a fork in the road and you see two inhabitant there, Alice and Bob. You want to figure out how to get to the airport. The following exchange occurs.

   You: I would like to go the airport.

   Alice: The airport is in the mountains or the road to the right goes to the airport.

   Bob: The airport is in the mountains and the road to the right goes to the airport.

   Alice: Bob is liar.

   Bob: The road to the right goes to the airport or the airport is not in the mountains.

   Which way is the airport, left or right?

6. Give your own example, different from the ones in class, of a predicate $P(x, y)$ such that

$$\forall x, \exists y, P(x, y)$$

   and

$$\exists y, \forall x, P(x, y)$$

   mean different things. Explain what each version means. (Each student in the class should have a different answer.)

7. Write the negation of each of these sentences.

   (a) Every time you roll a "6", you have to take a card.
   (b) There is a day in your life better than every other day.
   (c) In every good book, there is a plot twist or surprise ending.
   (d) Every math course has a topic that everyone finds easy to do.

8. Let $Q(a, b) =$ "$a - b = 2ab$" and assume for the rest of this question that a and b are always rational numbers (that means they are any fraction p/q where p and q are positive or negative whole numbers). Which of the following are true? Justify your answers, stating explicitly whether you're justifying by giving a single example, or by stating something for all cases.

   (a) $Q(2, 1)$
   (b) $\exists x, Q(x, 0)$
   (c) $\forall x, Q(x, 0)$
   (d) $\exists y, Q(y, y)$
   (e) $\forall x, \exists y, Q(x, y)$
   (f) $\exists x, \forall y, Q(x, y)$

9. Form a predicate and a quantified statement that represents the following sentence: "There is a Marquette student who gets A's in all of her classes."

10. Express each of the following statements using predicates and quantifiers. (You do not need to prove them!)

    (a) If n is a multiple of 5, then n ends in 5 or n ends in 0.

    (b) If n is not a multiple of 3, then $n^2 - 1$ is a multiple of 3

    (c) For all odd integers a and b, there is no real number x such that $x^2 + ax + b = 0$.

    (d) For every real number y, if $y \geq 0$, then there exists $x \in \mathbb{R}$ such that $x^2 = y$.

11. For parts (a) and (b) in the previous question, write the converse, inverse, and contrapositive. (Be sure to label which is which.)