

# MATH 2100 / 2105 / 2350 – HOMEWORK 8

due Thursday, **November 1**, at the beginning of class

*This homework assignment was written in L<sup>A</sup>T<sub>E</sub>X. You can find the source code on the course website.*

**Instructions:** This assignment is due at the *beginning* of class. **Staple your work** together (do not just fold over the corner). Please write the questions in the correct order. If I cannot read your handwriting, you won't receive credit. Explain all reasoning.

1. Prove by induction that for all positive integers  $n \geq 4$ ,

$$n! > 2^n.$$

2. Prove by induction that for all positive integers  $n$ , the number  $5^{2n+1} + 2^{2n+1}$  is divisible by 7.
3. Prove by induction that for all positive **odd** integers  $m$ , the number  $m^2 - 1$  is divisible by 8.
4. Prove by induction that for all positive integers  $n$ ,

$$1^3 + 2^3 + \cdots + n^3 = (1 + 2 + \cdots + n)^2.$$

You may use the theorem we proved from class that says

$$1 + 2 + \cdots + n = \frac{n(n+1)}{2}.$$

5. Prove by induction that for all positive integers  $n$ ,

$$\sum_{k=0}^n (k \cdot k!) = (n+1)! - 1.$$