MATH 2100 / 2105 / 2350 – HOMEWORK 7

due Thursday, **October 25**, at the beginning of class

This homework assignment was written in LaTeX. You can find the source code on the course website.

Instructions: This assignment is due at the *beginning* of class. **Staple your work** together (do not just fold over the corner). Please write the questions in the correct order. If I cannot read your handwriting, you won't receive credit. Explain all reasoning.

1. Prove that for any integer n, if n^2 is odd, then n is odd.
2. Suppose X, Y, and Z are integers. Prove that if X divides Y and Y divides Z, then X divides Z.
3. Decide if the following statement is true. If it is, prove it. If it's not, provide a counterexample.

   For all integers m, 3 divides $m^3 - m$.

4. Prove that all integers are rational numbers.
5. Decide if the following statement is true. If it is, prove it. If it's not, provide a counterexample.

   If n is a positive even integers, then $3^n + 1$ is divisible by 5.

6. Decide if the following statement is true. If it is, prove it. If it's not, provide a counterexample.

   If n is a positive even integer and $n \geq 4$ then $2^n - 1$ is not prime.