

MATH 2100 / 2105 / 2350 – HOMEWORK 6

due Thursday, October 11, at the beginning of class

This homework assignment was written in L^AT_EX. You can find the source code on the course website.

Instructions: This assignment is due at the *beginning* of class. **Staple your work** together (do not just fold over the corner). Please write the questions in the correct order. If I cannot read your handwriting, you won't receive credit. Explain all reasoning.

Mathematical Writing: An important component of this course is learning how to write mathematics correctly and concisely. Your goal should always be to convince the reader that you are correct! That means explaining your thinking and each step in your solution. We will talk more about this when we cover formal proofs in a few weeks, but for now I expect you to do the following: explain your reasoning, don't leave out steps, and use full sentences with correct spelling and grammar (including your use of math symbols). For example, don't write " $3 \in S \implies 3 \notin \bar{S}$ "; instead, write "Since $3 \in S$, it follows that $3 \notin \bar{S}$ ".

1. Read both of the handouts from class on Thursday (they're also posted on our website) and write yourself a page of bullet points for things to remember when writing proofs. Keep it somewhere safe, and bring it out every time you write a proof for the next few weeks so you can skim it over and make sure you've followed your bullet points. On your homework assignment, just tell me whether or not you did this, and include any thoughts you have about it. This one is on the honor system.
2. Complete the exercises we did in group work on Thursday again. These were #25, 26, 27 from Section 1.5. Be careful to choose the correct p and q !

3. Write a truth table for the proposition

$$(p \rightarrow q) \wedge (q \rightarrow p)$$

and explain the result in words, thinking about what this says about an implication and its converse.

4. Write a truth table to verify the logical equivalence

$$(p \rightarrow q) \wedge ((\neg q) \rightarrow (\neg p)) \equiv p \rightarrow q$$

and explain the result in words, thinking about what this says about an implication and its contrapositive.

5. In each of the following pairs, one of the statements implies the other. Decide *without using a truth table* which one implies the other. Do this by thinking about the meaning of both sides, and, as always, explain your reasoning.

(a) q $p \rightarrow q$

(b) $\forall y, \exists x, P(x, y)$ $\exists x, \forall y, P(x, y)$

(c) $s \wedge (p \rightarrow q) \wedge ((\neg p) \rightarrow q)$ $(q \vee r)$

(d) $(\forall x, P(x)) \vee (\forall x, \neg P(x))$ $(\exists x, P(x)) \rightarrow (\forall x, P(x))$