

Homework 5

(1) Let X be the set of US cities.

Let Y be the set of people living in the US.

Define $P(x,y) = y$ is the mayor of x .

(a) $\forall x \in X, \exists y \in Y, P(x,y)$ means "All cities have a person who is their mayor."

(b) $\exists y \in Y, \forall x \in X, P(x,y)$ means "There is a person who is the mayor of all cities."

(2) Negation of (a):

$$\neg (\forall x \in X, \exists y \in Y, P(x,y)) = \exists x \in X, \forall y \in Y, \neg P(x,y)$$

which means

there is a city such that nobody is its mayor.

(such that for all people p , p is not the mayor of the city)

Negation of (b):

$$\neg (\exists y \in Y, \forall x \in X, P(x,y)) = \forall y \in Y, \exists x \in X, \neg P(x,y)$$

Which means:

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For all people, there is a city that they're not the mayor of.

(3) [graded Q]

(4) Let M be the set of all Marquette students.
Let C be the set of all courses at Marquette.

Let $P(m, c) =$ "Student m is either not in course c or gets an A in course c ."

$$\boxed{\exists m \in M, \forall c \in C, P(m, c)}$$

(5) Let M be the set of all men.

Let $P(m) =$ " m is an island ".

"No man is an island" = $\boxed{\forall m \in M, \neg P(m)}$

(6) [graded Q]