MATH 2100 / 2105 / 2350 – HOMEWORK 5

due Thursday, October 4, at the beginning of class

This homework assignment was written in LaTeX. You can find the source code on the course website.

Instructions: This assignment is due at the *beginning* of class. **Staple your work** together (do not just fold over the corner). Please write the questions in the correct order. If I cannot read your handwriting, you won't receive credit. Explain all reasoning.

Mathematical Writing: An important component of this course is learning how to write mathematics correctly and concisely. Your goal should always be the convince the reader that you are correct! That means explaining your thinking and each step in your solution. We will talk more about this when we cover formal proofs in a few weeks, but for now I expect you to do the following: explain your reasoning, don't leave out steps, and use full sentences with correct spelling and grammar (including your use of math symbols). For example, don't write "$3 \in S \implies 3 \notin \overline{S}$"; instead, write "Since $3 \in S$, it follows that $3 \notin \overline{S}$".

1. Give your own example (different from the ones in class) of a predicate $P(x, y)$ where

$$\forall x, \exists y, P(x, y)$$

and

$$\exists y, \forall x, P(x, y)$$

mean different things. Explain what each version means. (Every student in the class should have a different answer.)

2. Write the negation of both of your examples from Exercise 1, both symbolically (with $\forall$, $\exists$, and P), and in English.

3. Form a predicate and a quantified statement that represents the following sentence: "Every university has a dorm that is at least 20 years old.".

4. Form a predicate and a quantified statement that represents the following sentence: "There is a Marquette student who gets A's in all of her classes."

5. Form a predicate and a quantified statement that represents the following sentence: "No man is an island."

6. Let $Q(a, b) =$ "$a + 2b = ab$", and assume for the rest of this question that a and b are always integers. Which of the following are true? Justify your answers, stating explicitly whether you're justifying by giving a single example, or by stating something for all cases.

   (a) $Q(2, 1)$
   (b) $\exists x, Q(x, 0)$
   (c) $\forall x, Q(x, 0)$
   (d) $\exists y, Q(y, y)$
   (e) $\forall x, \exists y, Q(x, y)$
   (f) $\exists x, \forall y, Q(x, y)$