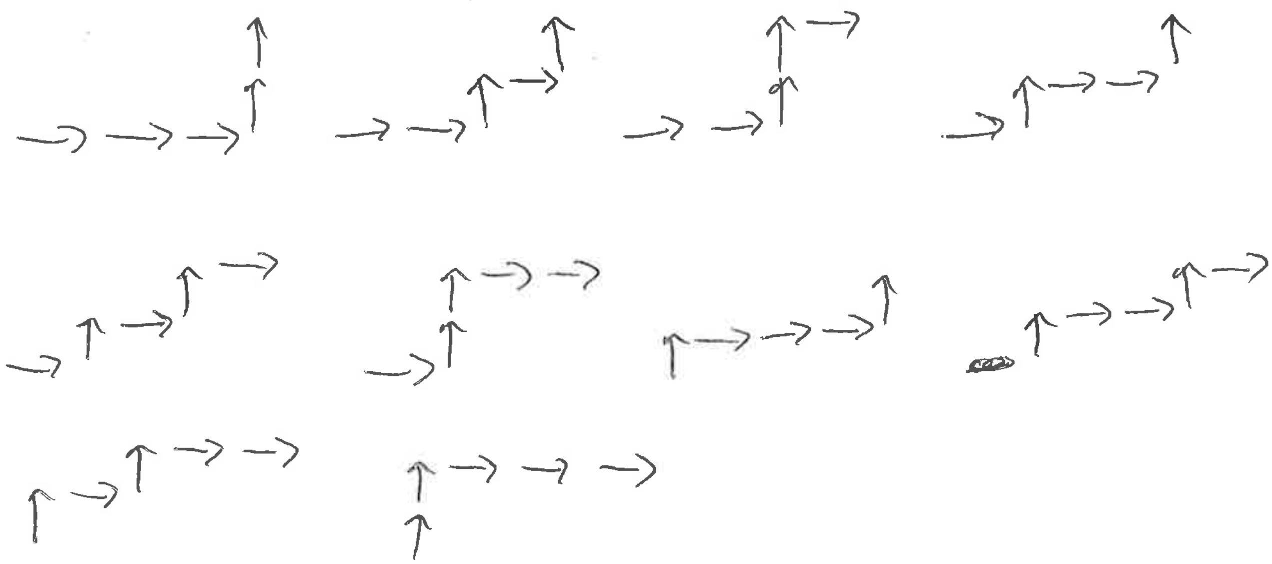
Homework 3

1) There are 10 paths.



2) [Graded Question]

3) [Graded Question]

4) Method 1:

pick 1st 5 people, pick next 5 people, pick next 5, pick next 5, pick last 5

$$\frac{\binom{5N}{5}\binom{5N-5}{5}\binom{5N-10}{5}\binom{5N-15}{5}\cdots\binom{10}{5}\binom{5}{5}}{N!}$$

The numerator put the teams in order, but the order shouldn't matter, so divide by N! to forget the order.

Method 2:

Put all $5N$ people in a line ($(5N)!$ ways)

D J A E C B F H I G .....

put first 5 on Team 1, second 5 on Team 2, etc. But now you need to forget the order of each team ($5!$ for team 1, $5!$ for team 2, $5!$ for team 3, etc)

and the order of the teams ($N!$)

So, you get $$\frac{(5N)!}{(5!)^N (N!)}$$

(looks different than Method 1, but they are the same).

5) First put the $5N$ people in teams like Q4, then pick an opponent for Team 1 ($N-1$ choices), then one for Team 2 (if not already picked) ($N-3$ choices), etc.

So, the answer is

$$\left(\text{answer for Q4, either form}\right) \cdot (N-1)(N-3)(N-5)\cdots(3)(1).$$

(We need to assume N is even for this to work!)

Challenge: Start with the idea of Method 2 above and see if you can get $\dfrac{(5N)!}{(5!)^N\left(\frac{N}{2}\right)!\,2^{N/2}}$

6) Coefficient of x^{37} in $x^4(x+2)^{50}$.

First observe that this is equal to the coefficient of x^{33} in $(x+2)^{50}$.

Binomial Theorem with $n=50$ and $y=2$:

$$(x+2)^{50} = \sum_{k=0}^{50} C(50,k)\cdot x^k \cdot 2^{50-k}$$

x^{33} is when $k=33$. This term is

$$\boxed{C(50,33)\cdot 2^{17}}.$$