MATH 2100 / 2105 / 2350 – HOMEWORK 3

due Thursday, September 20, at the beginning of class

This homework assignment was written in LaTeX. You can find the source code on the course website.

Instructions: This assignment is due at the *beginning* of class. Staple your work together (do not just fold over the corner). Please write the questions in the correct order. If I cannot read your handwriting, you won't receive credit.

Mathematical Writing: An important component of this course is learning how to write mathematics correctly and concisely. Your goal should always be the convince the reader that you are correct! That means explaining your thinking and each step in your solution. We will talk more about this when we cover formal proofs in a few weeks, but for now I expect you to do the following: explain your reasoning, don't leave out steps, and use full sentences with correct spelling and grammar (including your use of math symbols). For example, don't write "$3 \in S \implies 3 \notin \overline{S}$"; instead, write "Since $3 \in S$, it follows that $3 \notin \overline{S}$".

1. Suppose that you start at an intersection, and need to walk 3 blocks east and 2 blocks north, while only moving one block east or north at a time and never doubling back (never moving west or south). Draw all the possible paths that let you move 3 blocks east and 2 blocks north.

2. Suppose that you start at an intersection, and need to walk N blocks east and M blocks north, subject to the same constraints as the previous question. How many ways are there to do this? (*Hints:* All possible paths have the same number of total steps. How many? If you were to write out a list of your moves, like "EENENN", what do you have to choose in each case?)

3. If a basketball team of twelve people has five guards, four forwards, and three centers, then in how many ways can the coach pick two guards, two forwards, and one center to play in a game.

4. How many ways can you take $5N$ members of a basketball club and split them into N teams of 5 people each? (The order of the people in the team doesn't matter, and the order of the teams themselves doesn't matter.)

5. How many ways can you take $5N$ members of a basketball club, split them into N teams of 5 people each, and then pick pairs of teams to play each other head-to-head?

6. Without using a computer or doing it by hand, what is the coefficient of x^{37} in the polynomial $x^4(x+2)^{50}$?