MATH 2100 / 2105 / 2350 – HOMEWORK 2

due Thursday, September 13, at the beginning of class

This homework assignment was written in LaTeX. You can find the source code on the course website.

Instructions: This assignment is due at the *beginning* of class. Staple your work together (do not just fold over the corner). Please write the questions in the correct order. If I cannot read your handwriting, you won't receive credit.

Mathematical Writing: An important component of this course is learning how to write mathematics correctly and concisely. Your goal should always be the convince the reader that you are correct! That means explaining your thinking and each step in your solution. We will talk more about this when we cover formal proofs in a few weeks, but for now I expect you to do the following: explain your reasoning, don't leave out steps, and use full sentences with correct spelling and grammar (including your use of math symbols). For example, don't write "$3 \in S \implies 3 \notin \overline{S}$"; instead, write "Since $3 \in S$, it follows that $3 \notin \overline{S}$".

1. Use a Venn diagram to determine whether each of the following set equality is true or false. If true, explain why (use your Venn diagram—I'm not asking for a formal proof yet); if false, give examples of sets for which the two sides are not equal.
$$A \cap (B \cup C) = (A \cap B) \cup C$$

2. Determine whether the statement below is true or false. If true, give a few sentences of justification (not a formal proof). If false, give specific examples of sets for which the two sides are not equal.

   For all sets A, B, and C: if $B \subseteq C$, then $A \times B \subseteq A \times C$.

3. Determine whether the statement below is true or false. If true, give a few sentences of justification (not a formal proof). If false, give specific examples of sets for which the two sides are not equal.

   For all sets A, B, and C: $(A \cup B) \times (A \smallsetminus B) = A^2 \smallsetminus B^2$.

4. In New Hampshire, license plates contain exactly 8 letters with all of the following properties:

   i) The first and third letters are vowels.

   ii) All other letters are consonants.

   iii) No letter is repeated more than once.

   iv) The letter "Y" is considered a consonant, not a vowel.

   v) The license plate does end in "X".

   How many different license plates can be made that obey these rules?

5. You roll a six-sided die five times. How many possible outcome sequences are there in which you do not roll the same number twice in a row? For example, 14146 and 64141 both count, but 11464 does not.

6. A six-digit number is a number between 100000 and 999999. How many six digit numbers are there in which either all of the digits are even, or all of the digits are odd? For example, 517931 counts, but 214365 does not, and 022446 is not even a valid six-digit number.