

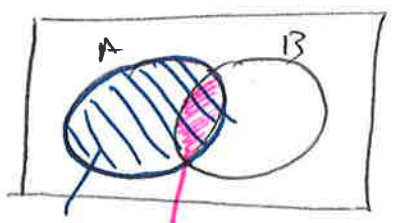
# Homework 1 Solutions

Math 2100/2103  
2350

Fall 2018 1

1)  $A \cup (A \cap B) \stackrel{?}{=} A$

a)



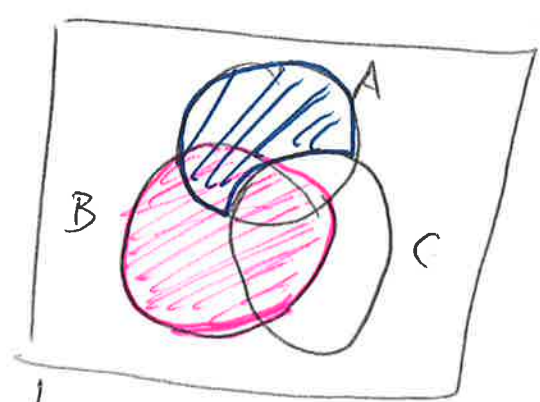
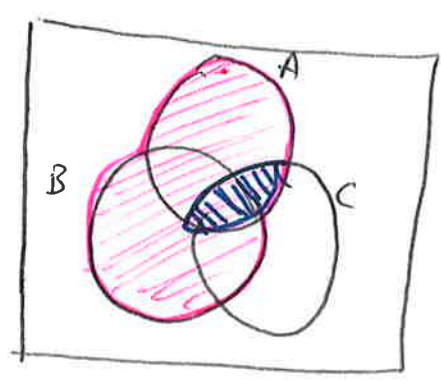
A       $A \cap B$

↓      ↓

$A \cup (A \cap B)$  is the region shaded by either blue or pink, which is clearly the same as A.

So, TRUE

b)  $(A \cup B) \setminus (A \cap C) \stackrel{?}{=} B \cup (A \setminus C)$



●  $\setminus$  ● is everything that is pink, but not blue.

●  $\cup$  ● is everything that is pink or blue

Not equal because  $A \cap B \cap C$  is included on the right, but excluded on the left.

we have to give an example. Keep it simple and 2  
use your Venn Diagram to help.

$$A = \{1\}, B = \{1\}, C = \{1\}$$

$$\text{Then } (A \cup B) - (A \cap C) = \{1\} - \{1\} = \emptyset$$

$$\begin{aligned} B \cup (A \cap C) &= \{1\} \cup (\{1\} \cap \{1\}) \\ &= \{1\} \cup \{1\} = \{1\} \end{aligned}$$

So, FALSE.

$$2) a) S = \left\{ \frac{n}{d} : n \in \mathbb{Z}, d \in \mathbb{Z}, n-d = 2k \text{ for some integer } k \right\}$$

Note: To be more precise, I should have said something like "the numerator and denominator in reduced form", but don't worry about that.

$$b) T = \left\{ n \in \mathbb{N} : n > 10 \text{ and } n = 100k + 50 + \ell \text{ where } k \in \mathbb{N}, \ell \in \mathbb{N}, \text{ and } 0 \leq \ell \leq 9 \right\}$$

Wrong answer:  $\left\{ n \in \mathbb{N} : n > 10 \text{ and the tens digit of } n \text{ is } 5 \right\}$

$$c) R = \{ (a, b) \in \mathbb{N} \times \mathbb{N} : a > 0, b > 0, a + b = 3k \text{ for some integer } k \}$$

$$3) a) A = \{ x \in \mathbb{R} : x^2 \in \mathbb{N} \}$$

This is the set of ~~all~~ real #s whose squares are non-negative whole #s. Some examples:  $-3, 0, \sqrt{2}, \sqrt{3}, 5$

$$b) B = \{ S \subseteq \{1, 2, 3, 4\} : \text{the sum of the elements in } S \text{ is even} \}$$

B is the set of subsets of  $\{1, 2, 3, 4\}$  whose sum is even. All the elements of B are

$$B = \{ \emptyset, \{2\}, \{4\}, \{1, 3\}, \{2, 4\}, \{1, 2, 3\}, \{1, 3, 4\}, \{1, 2, 3, 4\} \}$$

$$c) C = \{ q \in \mathbb{N} : q = 2k \text{ for some } k \in \mathbb{N} \text{ and } q = 2l + 1 \text{ for some } l \in \mathbb{N} \}$$

This translates to "all natural #s that are even ( $q = 2k$ ), and odd ( $q = 2l + 1$ )"

This set is empty.

$$4) X = \{2x : x \in \mathbb{Z}\}, Y = \{3y : y \in \mathbb{Z}\}$$

$$a) X \cap Y = \{6x : x \in \mathbb{Z}\}$$

(if a # is a multiple of 2 and of 3,  
then it's a multiple of 6)

$$b) X - Y = \text{the multiples of 2 that aren't multiples of 3} \\ = \{2x : x \in \mathbb{Z} \text{ and } 2x \neq 3y \text{ for some } y \in \mathbb{Z}\}$$

$$c) Y - X = \{3y : y \in \mathbb{Z} \text{ and } 3y \neq 2x \text{ for some } x \in \mathbb{Z}\}$$

$$d) \mathbb{Z} - X = \text{integers that are not even} \\ = \{2k+1 : k \in \mathbb{Z}\}$$

$$5) N = \{1, 2, \dots, n\}$$

$$a) |N \times N| = |N| \cdot |N| = n \cdot n = n^2$$

$$b) \{(a, b) \in N \times N : a \neq b\} \\ = (N \times N) \setminus \{(k, k) : k \in N\}$$

Size is  $n^2 - n$

c) complicated, don't worry about this one