

# MATH 2100 / 2105 / 2350 – HOMEWORK 11

due Thursday, **November 29**, at the beginning of class

*This homework assignment was written in L<sup>A</sup>T<sub>E</sub>X. You can find the source code on the course website.*

**Instructions:** This assignment is due at the *beginning* of class. **Staple your work** together (do not just fold over the corner). Please write the questions in the correct order. If I cannot read your handwriting, you won't receive credit. Explain all reasoning.

1. Let  $f : \mathcal{P}(\{1, 2, 3, 4\}) \rightarrow \mathcal{P}(\{1, 2, 3\})$  be defined by  $f(A) = A \setminus \{4\}$ . Draw the arrow diagram for the function. Determine whether or not it's injective, surjective, and bijective. Make sure to justify your answers (either with the arrow diagram, or a formal proof).
2. Let  $A = \{0, 1, 2, 3\}$  and let  $B = \{000, 001, 010, 011, 100, 101, 110, 111\}$  be the set of binary strings with three digits. Define  $g : B \rightarrow A$  by  $g(s) = [\text{the number of 1s in } s]$ . Draw the arrow diagram for the function. Determine whether or not it's injective, surjective, and bijective. Make sure to justify your answers (either with the arrow diagram, or a formal proof).
3. Let  $c : \mathcal{P}(\{x, y, z\}) \rightarrow \mathcal{P}(\{x, y, z\})$  be the function with the rule  $c(A) = \{x, y, z\} \setminus A$ , and let  $n : \mathcal{P}(\{x, y, z\}) \rightarrow \{0, 1, 2, 3\}$  be the function such that  $n(A)$  is the number of elements in the set  $A$ . Which composition makes sense,  $c \circ n$  or  $n \circ c$ ? For the one that is defined, give the domain, codomain, range, and draw the arrow diagram.
4. Prove that the function  $h : \mathbb{N} \rightarrow \mathbb{N}$  defined by  $h(n) = [\text{the sum of the digits in } n \text{ (in base 10)}]$  is surjective. Prove that it's not injective.
5. Let  $h : [2, \infty) \rightarrow (0, 1]$  be the function with the rule  $h(x) = \frac{1}{x-1}$ . Prove that  $h$  is a bijection by proving it is injective and surjective. Then compute  $h^{-1}(x)$  and give its domain, codomain, and range.