

MATH 2100 / 2105 / 2350 – HOMEWORK 11

due Thursday, **November 29**, at the beginning of class

This homework assignment was written in L^AT_EX. You can find the source code on the course website.

Instructions: This assignment is due at the *beginning* of class. **Staple your work** together (do not just fold over the corner). Please write the questions in the correct order. If I cannot read your handwriting, you won't receive credit. Explain all reasoning.

1. Let $f : \mathcal{P}(\{1, 2, 3, 4\}) \rightarrow \mathcal{P}(\{1, 2, 3\})$ be defined by $f(A) = A \setminus \{4\}$. Draw the arrow diagram for the function. Determine whether or not it's injective, surjective, and bijective. Make sure to justify your answers (either with the arrow diagram, or a formal proof).
2. Let $A = \{0, 1, 2, 3\}$ and let $B = \{000, 001, 010, 011, 100, 101, 110, 111\}$ be the set of binary strings with three digits. Define $g : B \rightarrow A$ by $g(s) = [\text{the number of 1s in } s]$. Draw the arrow diagram for the function. Determine whether or not it's injective, surjective, and bijective. Make sure to justify your answers (either with the arrow diagram, or a formal proof).
3. Let $c : \mathcal{P}(\{x, y, z\}) \rightarrow \mathcal{P}(\{x, y, z\})$ be the function with the rule $c(A) = \{x, y, z\} \setminus A$, and let $n : \mathcal{P}(\{x, y, z\}) \rightarrow \{0, 1, 2, 3\}$ be the function such that $n(A)$ is the number of elements in the set A . Which composition makes sense, $c \circ n$ or $n \circ c$? For the one that is defined, give the domain, codomain, range, and draw the arrow diagram.
4. Prove that the function $h : \mathbb{N} \rightarrow \mathbb{N}$ defined by $h(n) = [\text{the sum of the digits in } n \text{ (in base 10)}]$ is surjective. Prove that it's not injective.
5. Let $h : [2, \infty) \rightarrow (0, 1]$ be the function with the rule $h(x) = \frac{1}{x-1}$. Prove that h is a bijection by proving it is injective and surjective. Then compute $h^{-1}(x)$ and give its domain, codomain, and range.