

MATH 2100 / 2105 / 2350 – HOMEWORK 1

due Thursday, September 6, at the beginning of class

This homework assignment was written in L^AT_EX. You can find the source code on the course website.

Instructions: This assignment is due at the *beginning* of class. Staple your work together (do not just fold over the corner). Please write the questions in the correct order. If I cannot read your handwriting, you won't receive credit.

Mathematical Writing: An important component of this course is learning how to write mathematics correctly and concisely. Your goal should always be to convince the reader that you are correct! That means explaining your thinking and each step in your solution. We will talk more about this when we cover formal proofs in a few weeks, but for now I expect you to do the following: explain your reasoning, don't leave out steps, and use full sentences with correct spelling and grammar (including your use of math symbols). For example, don't write " $3 \in S \implies 3 \notin \bar{S}$ "; instead, write "Since $3 \in S$, it follows that $3 \notin \bar{S}$ ".

1. Use a Venn diagram to determine whether each of the following set equalities is true or false. If true, explain why (use your Venn diagram—I'm not asking for a formal proof yet); if false, give examples of sets for which the two sides are not equal.
 - (a) $A \cup (A \cap B) = A$
 - (b) $(A \cup B) \setminus (A \cap C) = B \cup (A \setminus C)$
2. Write each of the following sets in set-builder notation.
 - (a) The set S of rational numbers such that the difference between the numerator and the denominator is even.
 - (b) The set T of positive integers that are bigger than 10 and whose tens digit is a 5.
 - (c) The set R of pairs (a, b) of positive integers whose sum is a multiple of 3.
3. List five elements in each of the following sets, unless there are fewer than 5 elements in the set (in which case, justify how you know you've listed all of the elements).
 - (a) $A = \{x \in \mathbb{R} : x^2 \in \mathbb{N}\}$
 - (b) $B = \{S \subseteq \{1, 2, 3, 4\} : \text{the sum of the elements of } S \text{ is even}\}$
 - (c) $C = \{q \in \mathbb{N} : q = 2k \text{ for some } k \in \mathbb{N} \text{ and } q = 2\ell + 1 \text{ for some } \ell \in \mathbb{N}\}$
4. Define the sets $X = \{2x : x \in \mathbb{Z}\}$ and $Y = \{3y : y \in \mathbb{Z}\}$. Describe each of the following as simply as possible using set-builder notation.
 - (a) $X \cap Y$
 - (b) $X \setminus Y$
 - (c) $Y \setminus X$
 - (d) $\mathbb{Z} \setminus X$
5. Let n be a positive integer and define $N = \{1, 2, \dots, n\}$ to be the set of positive integers from 1 to n . For example, if $n = 4$, then $N = \{1, 2, 3, 4\}$. Each of the following answers should have the variable n in it.
 - (a) What is the size of the set $N \times N$?
 - (b) What is the size of the set $\{(a, b) \in N \times N : a \neq b\}$?
 - (c) What is the size of the set $\{(a, b, c) \in N \times N \times N : a \neq b, a \neq c, b \neq c\}$?