MATH 2100 / 2105 / 2350 – EXAM 2

November 15, 2018

Name: ______________________________

Instructions: Please write your work neatly and clearly. **You must explain all reasoning. It is not sufficient to just write the correct answer.** You have 75 minutes to complete this exam. You may not use calculators, notes, or any other external resources.

Scores

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1. Let A and B be sets. Define the following things. You must give both their names and their definitions.

   (a) $A \cap B$

   (b) $A \times B$

   (c) $\mathcal{P}(A)$

2. Prove that for any positive integer n, if n^2 is odd then n is odd.

3. Define the predicate $Z(a, b, c) =$ "$a^2 + b^2 = c^2$". Determine whether each of the following are true, and justify your reasoning.

   (a) $\exists a \in \mathbb{N}, \exists b \in \mathbb{N}, \exists c \in \mathbb{N}, Z(a, b, c)$

   (b) $\forall a \in \mathbb{R}, \forall b \in \mathbb{R}, \exists c \in \mathbb{R}, Z(a, b, c)$

   (c) $\forall a \in \mathbb{Q}, Z(a, 2a, 3a)$

4. Prove by contradiction that there do not exist integers a and b such that $18a + 6b = 1$.

5. Prove or disprove that for any two sets A and B, $\mathcal{P}(A) \cup \mathcal{P}(B) \subseteq \mathcal{P}(A \cup B)$.

6. Prove that for all $n \geq 0$:

$$\sum_{i=0}^{n} i2^i = 2 + (n-1)2^{n+1}$$

7. Prove that if any 5 points are chosen from a circle of radius 2, then there exist a pair of points within distance $\sqrt{8}$ of each other.

8. Let $A_1, A_2, A_3, \ldots$ be an infinite collection of sets with the property that $|A_1| = 1$, $|A_2| = 2$, $|A_3| = 3$, etc. In general, $|A_i| = i$. Prove (probably by induction) that for any $n \geq 1$:

$$|A_1 \times A_2 \times \cdots \times A_n| = n!.$$

You may use the fact that $|A \times B| = |A| \cdot |B|$.

9. (a) Prove that if 3 divides $4^{n-1} - 1$ then 3 divides $4^n - 1$.

(b) What you proved in part (a) did not quite use induction, but it was close. Explain how your answer to part (a) would fit inside an induction proof of a more general fact.

10. (a) Prove that the implication $p \to q$ is logically equivalent to $\neg p \vee q$.

(b) The statement "If n ends in 5, then n is a multiple of 5" is written in the first form. Translate it to the second form.