MATH 2100 / 2105 / 2350 – EXAM 1

September 27, 2018

Name: Key

Instructions: Please write your work neatly and clearly. **You must explain all reasoning. It is not sufficient to just write the correct answer.** You have 75 minutes to complete this exam. You may not use calculators, notes, or any other external resources.

Scores

1	
2	
3	
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10	

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- To turn in work done specifically for the paper or assignment, and not to borrow work either from other students, or from assignments for other courses.
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If you understand and agree to abide by this honor code, sign here:

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1. Define E to be the set of even positive integers. Define T to be the set of positive multiples of 10.

   (a) Write E in set-builder notation. (Your answer should not include the English words "even positive integers"; use the appropriate mathematical version.)

   $E = \{2k : k \in \mathbb{Z}^+\}$

   (b) List five elements in the set $E \setminus T$.

   $E \setminus T = \{2, 4, 6, 8, 12, \dots\}$

   (c) Write the set $T \times E$ in set-builder notation.

   $T \times E = \{(10k, 2\ell) : k \in \mathbb{Z}^+, \ell \in \mathbb{Z}^+\}$

   (d) List five elements in the set $T \times E$.

   $(10,2), (10,4), (10,6), (10,8), (10,10)$

2. You are the coach of a dodgeball club with 32 members. According to the National Dodgeball League rulebook, a full dodgeball team consists of 8 players. You would like to split your club into four teams with 8 people each, then pair up the four teams into two pairs to play against each other. In how many ways can this be done?

Pick first group of 8: $C(32,8)$

Pick second - - - - - - -: $C(24,8)$

Pick third - - - - - - -: $C(16,8)$

Pick fourth - - - - - - -: $C(8,8)$

Then "forget" the order of the 4 teams: $\frac{1}{4!}$

Then pair off: 3 (AB/CD, AC/BD, AD/BC)

So: $\frac{C(32,8)\cdot C(24,8)\cdot C(16,8)\cdot C(8,8)}{4!}\cdot 3$

Another way: Order everyone: $32!$

Declare that first 8 people are a team

second 8 people are a team

third - - - - - - -

fourth - - - - - - -

"forget" the order of the 8 players in each team: $\frac{1}{(8!)^4}$

"forget" the order of the teams $\frac{1}{4!}$

pair off: 3

$= \frac{32!}{(8!)^4 4!}\cdot 3$

3. Determine whether the following statement is true or false. If true, justify why. If false, give explicit examples of sets that demonstrate it's false.

For all sets A and B: $|A \setminus B| = |A| - |B|$.

The statement is FALSE. Here's a counterexample.

If $A = \{1\}$ and $B = \{2\}$, then

$|A| = 1$, $|B| = 1$

$A \setminus B = \{1\} \setminus \{2\} = \{1\}$, so, $|A \setminus B| = 1$,

but $|A| - |B| = 1 - 1 = 0$.

4. (a) State the meaning of the notation $C(n,k)$. (It's not sufficient to just write the name. What does it signify / represent? What is its purpose?)

$C(n,k)$ is the ~~[illegible]~~ binomial coefficient.
It is the # of ways to choose a subset of k things from a set of n things, where order doesn't matter.

(b) Calculate $C(10,4)$. Simplify your answer so it's just a positive integer.

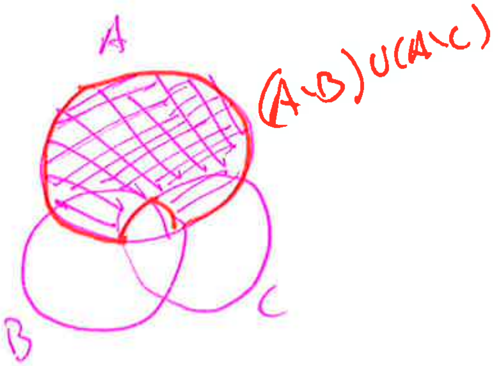
$$C(10,4) = \frac{10!}{4!(10-4)!} = \frac{10!}{4!\,6!} = \frac{10 \cdot \overset{3}{\cancel{9}} \cdot \cancel{8} \cdot 7 \cdot \cancel{6} \cdot \cancel{5} \cdot \cancel{4} \cdot \cancel{3} \cdot \cancel{2} \cdot \cancel{1}}{\cancel{4} \cdot \cancel{3} \cdot \cancel{2} \cdot \cancel{1} \cdot \cancel{6} \cdot \cancel{5} \cdot \cancel{4} \cdot \cancel{3} \cdot \cancel{2} \cdot \cancel{1}}$$

$$= 10 \cdot 3 \cdot 7 = \boxed{210}$$

5. Draw two Venn diagrams, one for each side of the equation below. Use your two drawings to determine whether the equation is true or false.

$$(A \setminus B) \cup (A \setminus C) = A \setminus (B \cap C)$$

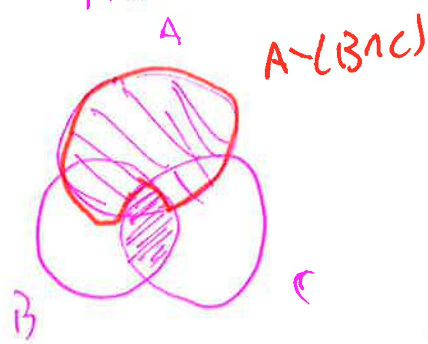
LHS:



$/\!/\!/\!/ = A \setminus B$

$\backslash\backslash\backslash\backslash = A \setminus C$

RHS



$/\!/\!/\!/\!/ = B \cap C$

$\backslash\backslash\backslash\backslash = A \setminus (\uparrow)$

According to the Venn Diagrams, the equation is <u>true</u>.

6. Create a truth table for the two sides of the equation below, and use it to determine whether the equation is true or false. **State clearly how you are using your truth table to come to your conclusion.**

$$\neg t \vee (s \wedge r) \vee r \equiv \neg(\neg r \wedge t)$$

r	s	t	$\neg t$	$s \wedge r$	$(\neg t) \vee (s \wedge r) \vee r$	$\neg r \wedge t$	$\neg(\ )$
T	T	T	F	T	T	F	T
T	T	F	T	T	T	F	T
T	F	T	F	F	T	F	T
T	F	F	T	F	T	F	T
F	T	T	F	F	F	T	F
F	T	F	T	F	T	F	T
F	F	T	F	F	F	T	F
F	F	F	T	F	T	F	T
					*		*

The two * columns are identical, so the truth table says that the logical equivalence is <u>true</u>.

7. This time you're the coach of a dodgeball club with 18 people: 10 women and 8 men. The official National Dodgeball League rulebook says that a coed team consists of 8 people, including at least one woman and at least one man[1]. How many ways can a team of 8 people with at least one woman and at least one man be chosen from a group of 10 women and 8 men?

Hint: How many ways can *any* team of 8 people be formed from 18? How many of those teams are "bad"? Use the answers to figure out how many are "good", i.e., satisfy the stipulated conditions.

All teams: $C(18,8)$

Bad teams: no men: $C(10,8)$

no women: $C(8,8)$

Good teams: $\boxed{C(18,8) - C(10,8) - \underbrace{C(8,8)}_{=1}}$

[1]Actually, it requires at least two women and at least two men, but I've simplified it for this question.

8. Recall that the Binomial Theorem says the following: For any positive integer n:

$$(x+y)^n = \sum_{k=0}^{n} C(n,k)x^k y^{n-k}.$$

Use the Binomial Theorem to find the coefficient of x^{15} in the polynomial $(x^3-1)^{10}$.

By the binomial theorem, with substitutions

$x \to x^3$

$y \to -1$

$n \to 10$,

we have

$$(x^3-1)^{10} = \sum_{k=0}^{10} C(10,k)\underbrace{(x^3)^k}_{=x^{3k}}(-1)^{n-k}$$

The ~~coefficient~~ term x^{15} occurs at the $k=5$ term:

$$C(10,5)(x^3)^5 = (-1)^{10-5}$$

$$= -C(10,5)$$

9. Seven couples go to the movies, and they find an empty row of 14 seats. How many ways are there for the seven couples to sit in the row such that everyone sits next to their partner?

Anybody can sit in the first seat: 14
Their partner must sit in the second: 1
Anybody remaining can sit in the third chair: 12
etc.

$\boxed{14 \cdot 12 \cdot 10 \cdot 8 \cdot 6 \cdot 4 \cdot 2}$

10. You find three people, Alice, Bob, and Charlie, who say the following:

Alice: At least one of us is telling the truth.

Bob: Alice is telling the truth and I am lying.

Charlie: Bob is telling the truth and I am lying.

Who is telling the truth and who is lying?

Let A = "Alice is telling the truth"
B = "Bob - - - - - - - - -"
C = "Charlie - - - - - - - - -"

P = "Alice's statement is true"
Q = "Bob's - - - - - - -"
R = "Charlie's - - - - - - -"

The truth table is:

A	B	C	P	Q	R	
T	T	T	T	F	F	
T	T	F	T	F	T	
T	F	T	T	T	F	
T	F	F	T	T	F	
F	T	T	T	F	F	
F	T	F	T	F	T	
F	F	T	T	F	F	
F	F	F	F	F	F	*

The only row for which
A ≡ P, B ≡ Q, C ≡ R
is when they're <u>all</u> <u>lying</u> –

FFF

11. **BONUS QUESTION. Do not attempt this question until you've finished all the others.**

How many ways can you split a class of $3n$ students into 3 groups that each have n students?

Group 1: $C(3n, n)$
Group 2: $C(2n, n)$
Group 3: $C(n, n) = 1$.
Now forget the order of the groups: (3 groups = 3! orders to forget)

$$\boxed{\frac{C(3n,n)\cdot C(2n,n)}{3!}}$$