MATH 13, FALL '16

HOMEWORK 9

Will not be graded, just for practice.

Write your answers neatly and clearly. Use complete sentences, and label any diagrams. List problems in numerical order and staple all pages together. Start each problem on a new page. Please show your work; no credit is given for solutions without work or justification.

Remember that you may discuss the problems with classmates, but all work should be your own. List the names of anybody with whom you discussed the problems at the top of the page.

1. Consider the cube with opposite corner points $(0,0,0)$ and $(2,2,2)$. Let $\boldsymbol{F} = \langle -y, 2yz, 1-z^2 \rangle$, and note that $\boldsymbol{F} = \text{curl}(\boldsymbol{A})$ for $\boldsymbol{A} = \langle yz^2, (yz+x), z^3 \rangle$.

    a) Let S be the surface of the cube (all six sides), with outward facing normal vectors. Calculate $\iint_S \boldsymbol{F}\, dS$.

    b) Let S be the surface consisting of all sides except the top, with outward facing normal vectors. Calculate $\iint_S \boldsymbol{F}\, dS$.

2. Let S be the surface consisting of the outside of the cone $z = \sqrt{x^2+y^2}$ bounded by the planes $z=1$ and $z=4$, with inward facing normal vectors. Note that S *does not* include the top nor bottom surfaces, just the outside of the cone. Let $\boldsymbol{F} = \left\langle x+y, z-x, \cos(e^{x^2-yz}) \right\rangle$. Calculate $\iint_S \text{curl}(\boldsymbol{F})\, dS$.

3. Let S be the surface of the solid bounded by the cylinder $x^2+y^2=1$ and the planes $z=-1$ and $z=2$ (so, S involves the outside of the cylinder as well as the top and bottom circular caps), with inward-facing normal vectors. Let $\boldsymbol{F} = \langle x^3, ze^x, 3zy^2 \rangle$. Find $\iint_S \boldsymbol{F} \cdot dS$.

4. Let W be the part of the solid ball $x^2+y^2+z^2 \leq 4$ with $z \geq 0$. Let $S = \partial W$ be the boundary surface of W, oriented with outward pointing normal vectors. The surface S consists of two smooth pieces: $S = S_1 + S_2$, where S_1 is the disk $x^2+y^2 \leq 4$, $z=0$; and S_2 is the hemisphere $x^2+y^2+z^2 = 4$, $z \geq 0$. Finally, we have a vector field

$$\mathbf{F} = \langle y^3 + z^3,\ x^5 + z^5,\ x^2 + y^2 \rangle$$

    a) What is the value of $\iint_S \mathbf{F} \cdot d\mathbf{S}$?

    b) What is the value of $\iint_{S_1} \mathbf{F} \cdot d\mathbf{S}$?

    c) What is the value of $\iint_{S_2} \mathbf{F} \cdot d\mathbf{S}$?