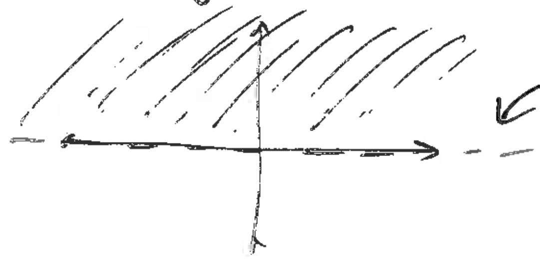
Q1: 3pts
Q2: 3pts
Q3: 2pts
Q4: 2pts

Math 13, Homework 1 Solutions

1. $f(x,y) = \cos\left(\frac{x}{\sqrt{y}}\right)$

(a) What is the domain? Sketch it.

Because of $\sqrt{y}$, we must have $y \geq 0$. However, since $\sqrt{y}$ is in the denominator, we can't have $y=0$. So, $y>0$. No restrictions on x.



[not including x-axis]

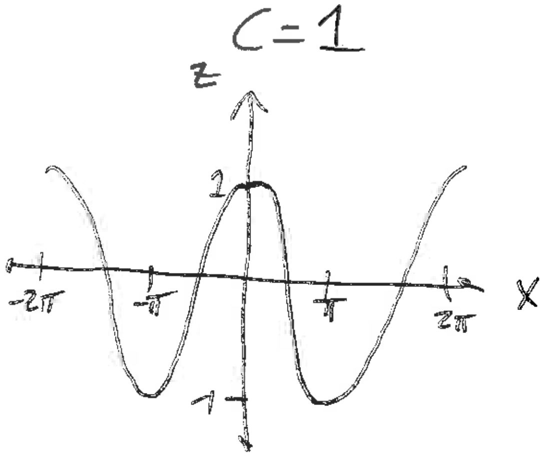
(b) What is the range of $f(x,y)$?

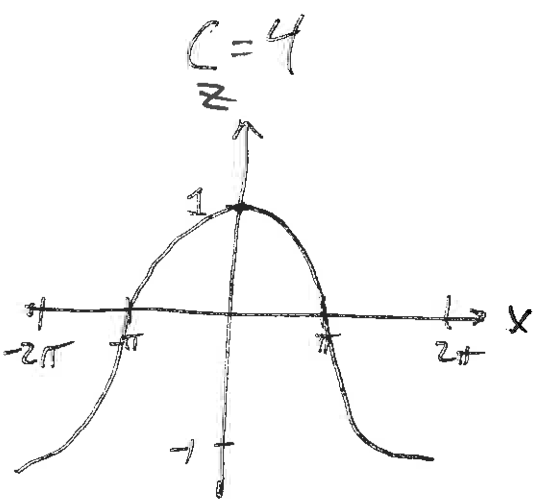
Recall that the range of the cosine function is $[-1,1]$. Therefore, the range of $f(x,y)$ is at most this big. The question is, can we manipulate x and y so that we can achieve every value in $[-1,1]$? The answer is yes. One way to do it is to fix $y=1$ and let x vary over all real #'s. This alone is enough to reach all of $[-1,1]$, even though we only used a single y-value.

(c) Describe the shape of vertical traces for fixed y-values.

Fix $y = C$, a constant. Then, the vertical trace at $y = C$ is the curve $z = \cos\left(\frac{x}{\sqrt{C}}\right) \cdot \sqrt{C}$ is just another constant, so let's call it D. The curve

$\cos\left(\frac{X}{D}\right)$ is like the curve $\cos(X)$, but stretched or shrunk horizontally. For example,





Try graphing some of the vertical traces for fixed x-values. Explain why they look the way they do.

3

2 (a) Calculate $\iint_R \frac{y}{2x+1}\,dA$, where $R = [0,3]\times[1,2]$.

Convert to iterated integral:

$$\int_{x=0}^{x=3}\int_{y=1}^{y=2} \frac{y}{2x+1}\,dy\,dx$$

Optional shortcut:

$$= \left(\int_{x=0}^{x=3} \frac{1}{2x+1}\,dx\right)\cdot\left(\int_{y=1}^{y=2} y\,dy\right)$$

$$= \left[\frac{1}{2}\ln(2x+1)\right]_{x=0}^{x=3}\cdot\left[\frac{y^2}{2}\right]_{y=1}^{y=2}$$

$$= \left(\frac{1}{2}\ln(7)-\frac{1}{2}\ln(1)\right)\left(2-\frac{1}{2}\right)$$

$$= \frac{1}{2}\ln(7)\cdot\frac{3}{2} = \boxed{\frac{3}{4}\ln(7)}$$

Without shortcut:

$$= \int_{x=0}^{x=3}\left[\frac{y^2}{2(2x+1)}\right]_{y=1}^{y=2} dx$$

$$= \int_{x=0}^{x=3}\left(\frac{2}{2x+1}-\frac{1}{2(2x+1)}\right)dx$$

$$= \frac{3}{2}\int_{x=0}^{x=3}\frac{1}{2x+1}\,dx$$

$$= \frac{3}{4}\left[\ln(2x+1)\right]_{x=0}^{x=3}$$

$$\boxed{= \frac{3}{4}\ln(7).}$$

(b) Explain without direct calculation why

$$\iint_S \frac{y}{2x+1}\, dA = 0$$

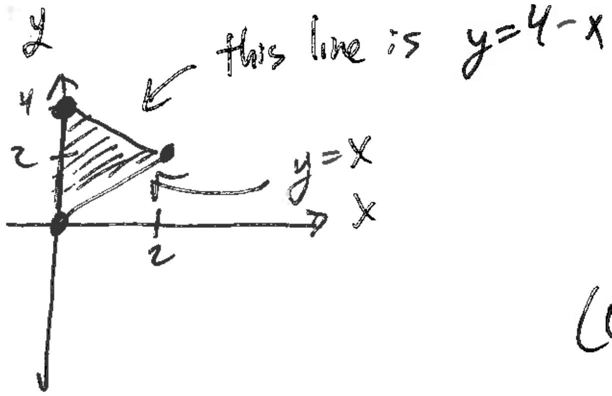
for $S = [1,2] \times [-1,1]$.

First, note that the function has symmetry around the y-values: the value at any positive $y = c$ and the corresponding value at $y = -c$ are the same, except with reverse signs.

Formally: $f(x,-y) = \frac{-y}{2x+1} = -\left(\frac{y}{2x+1}\right) = -f(x,y)$.

So, the integral over any rectangle that has symmetry in y-values (aka. symmetry over the x-axis) will be zero.

3. Integrate $h(x,y) = -2xy$ over the triangular region R in the xy-plane with corner points $(0,0)$, $(0,4)$, $(2,2)$.



Region: $\begin{cases} x \text{ in } [0,2] \\ y \text{ in } [x, 4-x] \end{cases}$

(Other order requires splitting into two regions)

$$\int_{x=0}^{x=2}\int_{y=x}^{y=4-x} -2xy\,dy = -2\int_{x=0}^{x=2}\left[\frac{xy^2}{2}\right]_{y=x}^{y=4-x} dx$$

$$= -\int_{x=0}^{x=2}\left(x(4-x)^2 - x(x)^2\right)dx = -\int_{x=0}^{x=2}\left(16x - 8x^2\right)dx$$

$$= -\left[8x^2 - \frac{8}{3}x^3\right]_{x=0}^{x=2} = -\left(8(4) - \frac{8}{3}(8)\right)$$

$$= -\left(32 - \frac{64}{3}\right)$$

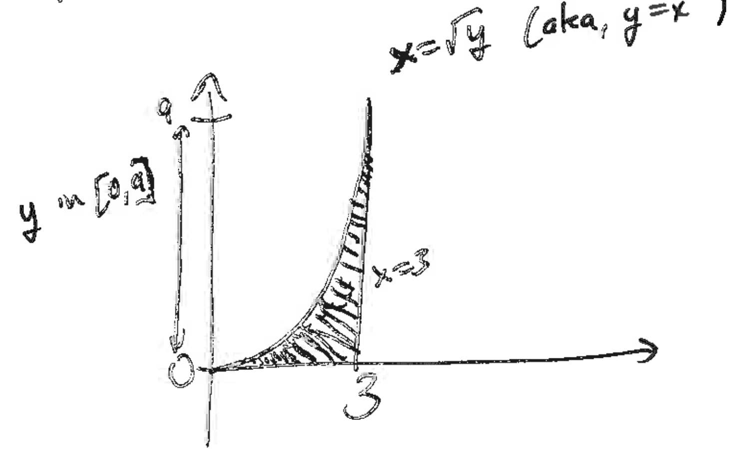
$$= -\frac{32}{3}$$

4. Rewrite the integral with the order of integration reversed. Do not evaluate.

$$\int_0^9 \int_{\sqrt{y}}^3 (y - \sqrt{x} + 1)\,dx\,dy.$$

The integrand doesn't matter.

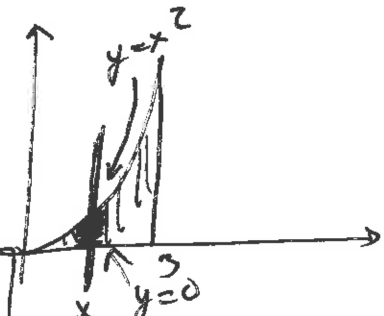
Step 1: Draw the region.



Step 2:

Describe in the other order:

$$\begin{cases} x \text{ in } [0,3] \\ y \text{ in } [0,x^2] \end{cases}$$



Answer: $$\boxed{\int_{x=0}^{x=3} \int_{y=0}^{y=x^2} (y - \sqrt{x} + 1)\,dy\,dx}$$