

NAME : _____

Math 13

Final Exam
November 20, 2016

Prof. Pantone

INSTRUCTIONS: This is a closed book exam and no notes are allowed. You are not to provide or receive help from any outside source during the exam except that you may ask the instructor for clarification of a problem. You have 180 minutes and you should attempt all problems.

- Print your name in the space provided.
- Calculators or other computing devices are not allowed.
- Except when indicated, you must show all work and give justification for your answer. **A correct answer with incorrect work will be considered wrong.**

All work on this exam should be completed in accordance with the Dartmouth Academic Honor Principle.

Helpful Hints:

- Use your time wisely. If you're worried about having enough time, start with the problems that are worth more points.
- Work carefully and methodically to prevent arithmetic errors.
- Work neatly so you have enough room.
- Don't Panic!

Problem	Points	Score
1	6	
2	32	
3	7	
4	7	
5	6	
6	6	
7	6	
8	0	
9	0	
Total	70	

Section 1: True/False.

1. (6) Choose the correct answer. *No justification is required for your answers. No partial credit will be awarded.*

- (a) The correction factor for polar integrals is the same as the correction factor for spherical integrals.

True

False

- (b) When computing the integral of a scalar function over a surface $\left(\iint_S f(x, y, z) dS\right)$, the surface S must have an orientation.

True

False

- (c) The vectors $\langle 3, 6, -1 \rangle$ and $\langle 2, 2, 18 \rangle$ are orthogonal.

True

False

- (d) For all vector fields \mathbf{F} , $\text{curl}(\text{div}(\mathbf{F})) = \mathbf{0}$.

True

False

- (e) Stokes' Theorem cannot be applied to surfaces S that are completely enclosed (i.e., no boundary).

True

False

- (f) The Divergence Theorem can only be applied to surfaces S that are completely enclosed (i.e., no boundary).

True

False

Section 2: Multiple Choice.

2. (32) Circle your answer. No justification is required. No partial credit will be awarded.

If it is not obvious which answer you have circled, you won't receive credit.

- (a) Let W be the three-dimensional solid that is a pyramid with a square base with corner points $(0, 0, 0)$, $(2, 0, 0)$, $(0, 2, 0)$, and $(2, 2, 0)$ and peak $(1, 1, 5)$. Let $\mathbf{F} = \langle x^2(y - e^z), -xy^2, 2xe^z \rangle$. Let S be the surface of W with outward-facing normal vectors. Calculate $\iint_S \mathbf{F} dS$.

(I) -1

(II) $-1/5$

(III) 0

(IV) $1/5$

(V) 1

- (b) Let $\mathbf{F} = \langle e^x \sin(y), e^x \cos(y) + x \rangle$. Let C be the boundary of the region between the graphs of $y = x^2$ and $y = 1$, oriented counterclockwise. Find $\int_C \mathbf{F} d\mathbf{r}$.

(I) 0

(II) 1

(III) $4/3$

(IV) $5/3$

(V) 2

(c) Let D be a region in the xy -plane with area equal to 7. Compute the surface area of the part of the plane $3x - y + 2z = 10$ lying over the region D .

(I) 7

(II) $\frac{14}{\sqrt{14}}$

(III) $\frac{7}{2}$

(IV) $\frac{\sqrt{14}}{2}$

(V) $\frac{7\sqrt{14}}{2}$

(d) The point $(2, 2, 5)$ in Cartesian coordinates is equal to $(r, \theta, z) = \underline{\hspace{2cm}}$ in cylindrical coordinates.

(I) $(\pi/8, \pi/4, \pi/6)$

(II) $(\sqrt{8}, \pi/4, 5)$

(III) $(2, \pi/2, 5)$

(IV) $(\sqrt{8}, \pi/2, 5)$

(V) $(\sqrt{2}, \pi/2, 5)$

(e) Calculate $\iint_R \frac{\cos(x)}{y} dA$ for $R = \left[0, \frac{\pi}{4}\right] \times [1, e^2]$.

(I) $\ln(2)$

(II) $\sqrt{2}$

(III) 2

(IV) 1

(V) $\ln(2) - 1$

(f) Let $\mathbf{F} = \langle yz + y^2ze^{xz}, xz + 2ye^{xz}, xy + xy^2e^{xy} \rangle$. Find a potential function for \mathbf{F} , if one exists.

(I) $f(x, y, z) = xyz + y^2e^{x+z}$

(II) $f(x, y, z) = xyz + y^2e^{xyz}$

(III) $f(x, y, z) = xyz + y^2e^{xz}$

(IV) $f(x, y, z) = xyz + xye^{xyz}$

(V) \mathbf{F} does not have a potential function.

(g) Which of these integrals computes the volume of a sphere of radius R ?

- (I) $\int_0^{2\pi} \int_0^{\pi/2} \int_0^R \rho^2 \sin(\phi) d\rho d\phi d\theta$
- (II) $\int_0^{2\pi} \int_0^{\pi/2} \int_0^R 1 d\rho d\phi d\theta$
- (III) $8 \int_{3\pi/2}^{2\pi} \int_0^{\pi/2} \int_0^R \rho^2 \cos(\phi) d\rho d\phi d\theta$
- (IV) $8 \int_{3\pi/2}^{2\pi} \int_0^{\pi/2} \int_0^R \rho^2 \sin(\phi) d\rho d\phi d\theta$
- (V) $\int_0^{2\pi} \int_0^{\pi} \int_0^R 1 d\rho d\phi d\theta$

(h) Use the Divergence Theorem to calculate the surface integral $\iint_S \langle x^3, y^3, z^3 \rangle dS$ where S is the surface of the sphere $x^2 + y^2 + z^2 = 1$ with outward-facing normal vectors.

- (I) 0
- (II) $\frac{12\pi}{5}$
- (III) $-\frac{6\pi}{5}$
- (IV) $\frac{6\pi}{5}$
- (V) None of the above

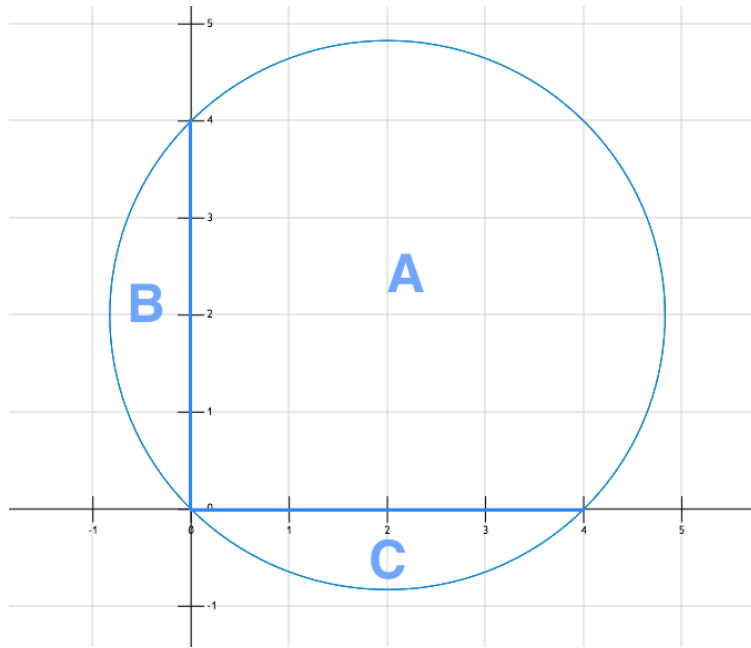
Section 3: Long Answer.

You must show all work to receive credit. If you need more space you may use the back of the page. You must clearly indicate on the front of the page that there is more work on the back of the page. Please work neatly and circle your final answer.

3. (7) Let C be the triangle with corner points $(1, 0, 0)$, $(0, 1, 0)$, and $(0, 0, 1)$, oriented clockwise when viewed from above. Let $\mathbf{F} = \langle x + y^2, y + z^2, z + x^2 \rangle$. Use Stokes' Theorem to calculate $\oint_C \mathbf{F} \, d\mathbf{r}$.

4. (7) Let W be the solid region bounded by $z = 0$ and the paraboloid $z = 1 - x^2 - y^2$. Let S be the boundary of W with outward-facing normal vectors. Let $\mathbf{F} = \langle y^2z + xy^2, xe^z + x^2y, ye^{yx} \rangle$. Compute $\iint_S \mathbf{F} \, d\mathbf{r}$.

5. (6) Consider the curve $r = 4(\cos(\theta) + \sin(\theta))$, graphed below for $0 \leq \theta \leq \pi$.



Let A be the region that lies inside the curve with $x \geq 0$ and $y \geq 0$. Let B be the region bounded that lies inside the curve with $x \leq 0$. Let C be the region that lies inside the curve with $y \leq 0$.

- (a) Find the area of the region A .

(b) Find the area of the region B .

(c) Find the area of the region C .

6. (6) Let R be the region bounded by the curves $xy = 1$, $xy = 2$, $xy^2 = 1$, and $xy^2 = 2$. Use the transformation $u = xy$ and $v = xy^2$ to calculate $\iint_{\mathcal{R}} y^2 dA$.

7. (6) Complete each question. (The two parts are unrelated.)

(a) Find the volume of the solid region bounded by the surfaces $z = y^2$ and $z = 1 - x^2$.

(b) Rewrite the integral with the order of integration reversed. *Do not evaluate the integral.*

$$\int_0^{16} \int_{\sqrt{y}}^4 (3xy + y^2) dx dy$$

For the problems on this page, you may use **Clairaut's Theorem** from Math 8.
Clairaut's Theorem: If $f(x, y)$ has second partial derivatives that exist and are continuous, then

$$\frac{\partial}{\partial x} \left(\frac{\partial f}{\partial y} \right) = \frac{\partial}{\partial y} \left(\frac{\partial f}{\partial x} \right).$$

In other words, the order in which you take the partial derivatives doesn't matter. The theorem holds for functions in three variables, too.

8. (0) **BONUS!** (3 points) Don't attempt this problem until you've finished the rest of the exam.

Let \mathbf{F} be any vector field with components whose second partial derivatives all exist and are continuous. Prove that

$$\operatorname{div}(\operatorname{curl}(\mathbf{F})) = 0.$$

9. (0) **BONUS!** (3 points) Don't attempt this problem until you've finished the rest of the exam.

Let $f(x, y, z)$ be any scalar function whose second partial derivatives all exist and are continuous. Prove that

$$\operatorname{curl}(\nabla f) = \mathbf{0}.$$