Your name:

Instructor (please circle):

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Math 11 Fall 2015, Homework 4, due Wed Oct 14

Please show your work. No credit is given for solutions without work or justification.

(1) Suppose you are hiking on terrain modeled by the function $z = \dfrac{10(x-2y)}{x^2+2y^2+1}$ and that you are at the point $(1, 0, 5)$. Assume that the positive y-axis points North and the positive x-axis points East.

(a) Determine the slope you would encounter if you traveled due South. What angle of inclination does this correspond to?

(b) Determine the slope you would encounter if you traveled due Northwest. What angle of inclination does this correspond to?

(c) Give a unit vector in the direction that you should travel if you want to take the route of steepest descent.

(2) The production P of corn in a given year depends on three main variables: average annual temperature T, average annual rainfall R, and seed cost S. Scientists estimate that average annual temperature is increasing at a rate of 0.1°C per year, average annual rainfall is decreasing at a rate of 0.05 cm per year, and average seed cost is increasing at a rate of \$0.05 per year. Farmers estimate that production is affected by temperature, rainfall, and seed cost in the following way:

$$\frac{\partial P}{\partial T} = -1 \qquad \frac{\partial P}{\partial R} = 2 \qquad \frac{\partial P}{\partial S} = -\frac{1}{2}.$$

(a) Explain in words what the ***signs*** of $\frac{\partial P}{\partial T}$, $\frac{\partial P}{\partial R}$, and $\frac{\partial P}{\partial S}$ mean.

(b) Use the chain rule to find the current rate of change in corn production $\frac{dP}{dt}$, where t is measured in years.

(3) Let $f(x, y) = x^3 + y^3 - 5xy + 4$.

(a) Find all critical points of $f(x, y)$.

(b) Use the second derivative test to determine whether $f(x, y)$ has a local maximum, a local minimum, or a saddle point at each critical point. If the second derivative test is inconclusive, state this.