Your name:

Instructor (please circle):

Craig Sutton Erik van Erp Jay Pantone Alex Barnett

Math 11 Fall 2015, Homework 1, due Wed Sep 23

Please show your work. No credit is given for solutions without work or justification.

(1) Sketch a picture of the pyramid with vertices

$$A = (0,0,0), B = (2,0,0), C = (2,2,0), D = (0,2,0), P = (1,1,4)$$

and answer the following questions.

(a) Derive a parametric equation for the straight line through points B and P.

(b) Use vector algebra to calculate the cosine of the angle BPA (i.e., the angle between line segments BP and AP at point P.)

(c) Use vector algebra to calculate the area of triangle BPA. (Hint: the area of a triangle is half of the area of a parallelogram.)

(2) For the vectors $\mathbf{v} = \langle 1, 0, -1 \rangle$ and $\mathbf{w} = \langle 0, 1, -1 \rangle$ calculate the components of the decomposition

$$\mathbf{w} = \mathbf{w}_{\parallel v} + \mathbf{w}_{\perp v}$$

where $\mathbf{w}_{\parallel v}$ is the projection of $\mathbf{w}$ along $\mathbf{v}$ while $\mathbf{w}_{\perp v}$ is the component of $\mathbf{w}$ perpendicular to $\mathbf{v}$.

(3) For each of the following equalities indicate whether it is true or false for all vectors $\mathbf{a}$ and $\mathbf{b}$ in $\mathbb{R}^3$. Use vector algebra to justify your answers.

(a) $(\mathbf{a}+\mathbf{b})\times(\mathbf{a}-\mathbf{b}) = \mathbf{a}\times\mathbf{a} - \mathbf{b}\times\mathbf{b}$

(b) $(\mathbf{a}+\mathbf{b})\cdot(\mathbf{a}-\mathbf{b}) = \mathbf{a}\cdot\mathbf{a} - \mathbf{b}\cdot\mathbf{b}$

(c) $\|\mathbf{a}+\mathbf{b}\|^2 = \|\mathbf{a}\|^2 + \|\mathbf{b}\|^2 + 2\mathbf{a}\cdot\mathbf{b}$.